The quantum vacuum at the foundation of classical optics

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MPL

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Maxwell's equations and a sum rule for elementary particles

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VACUUM

Warning!

VACUUM

Speculative!

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$$

Polarization of the vacuum

$$\mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M}$$

Magnetization of the vacuum

W. Heitler, "The quantum theory of radiation", p.113, 3rd edition, Oxford University 1954

We shall treat the polarization of the vacuum in detail in § 32, but the qualitative consequences are easily seen: A constant field merely leads to a constant polarization. The polarizability is field independent and thus all charges are changed by a universal constant factor (the 'dielectric constant' of the vacuum). Since there is no way of experimenting in an 'ideal vacuum' where this polarizability is absent, this effect is unobservable in principle (although the polarizability turns out to be infinite owing to the infinite number of pairs which contribute). An inhomogeneous field creates, in addition, a non-uniform polarization whose effects are observable (and finite). For example, an additional light wave will be scattered by the polarization dipoles, i.e. in the Coulomb field. Similarly, two light waves will be scattered by each other (§ 32).

Victor S. Weisskopf: Kongelige Danske Videnskabernes Selskab, Mathematisk-fysiske Meddelelser XIV, No.6, (1936)

Weisskopf's three problematic quantities

1.- ...
2.- ...
3.- A spatially and temporally constant and field independent electric and magnetic polarizability of the vacuum

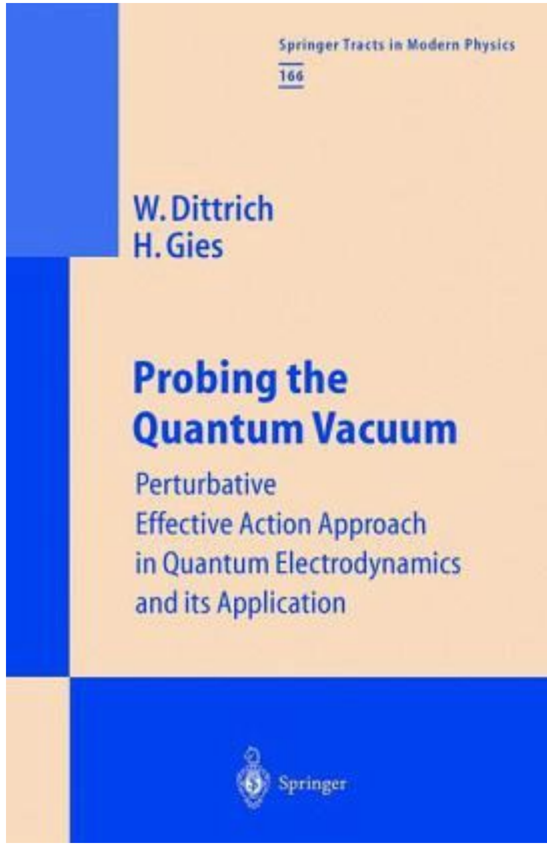
These quantities relate to the field free vacuum.
It can be taken as self-evident that they can have no physical meaning.

- Warning

$$c_{\rm rel} = \sqrt{\frac{E}{m}} \qquad\qquad c_{\rm light}$$

are not necessarily equivalent!

K. Scharnhorst, Phys. Lett. **B 236**, 354(1990)



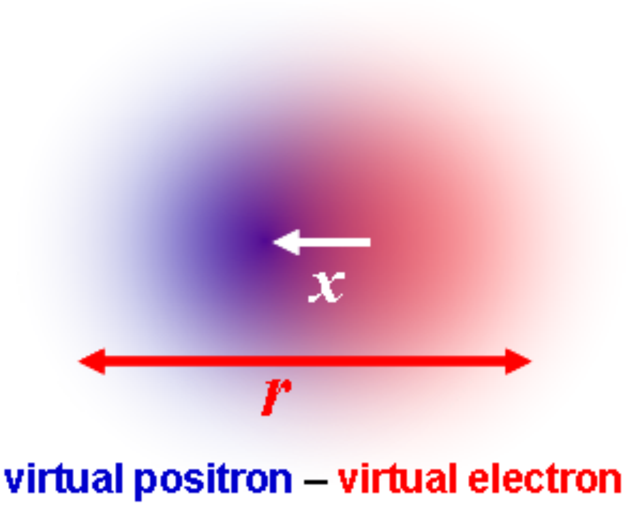
- We think of the vacuum as a dielectric and diamagnetic medium

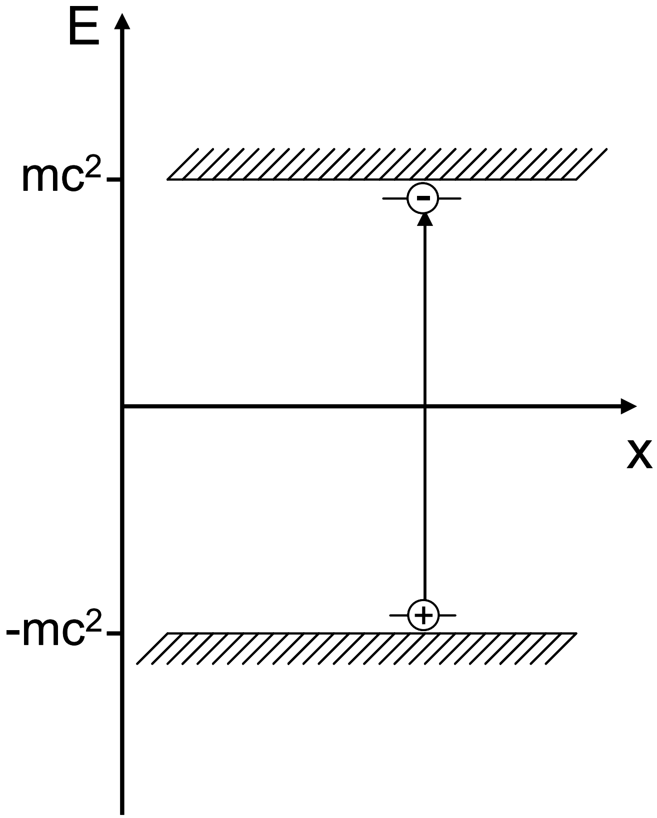
$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} \equiv \mathbf{P}_0 + \mathbf{P}$$

$$\mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M} \equiv \mathbf{M}_0 - \mathbf{M}$$

derive ε_0, μ_0 from properties of the vacuum

$$\text{polarization} = \frac{\text{dipole moment}}{\text{volume of dipole}}$$

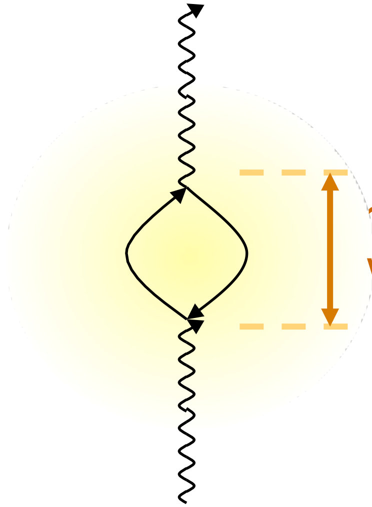




$$\hbar\omega_0 = 2mc_{\rm rel}^2$$

$$\langle d \rangle = e\langle x \rangle = \frac{e^2}{m\omega_0^2}\zeta E = \frac{e^2\hbar^2\zeta}{2m^3c_{\rm rel}^4}E$$

Transient creation of the dipole



$\dfrac{\hbar}{mc_{rel}}$

$$\Delta x\, \Delta p \geq \frac{\hbar}{2} \qquad \Delta p = mc_{\rm rel}$$

$$\Delta x \geq \frac{\hbar}{2mc_{\rm rel}}$$

$$V = \eta \left(\frac{\hbar}{mc_{\rm rel}} \right)^3$$

Critical dc electric field strength

$$eE_{\rm crit}\lambda_{\rm C} = mc_{\rm rel}^2$$

Bohr (before 1931), Sauter 1931, Schwinger 1951

$$P_0 = \frac{d}{V} = \frac{e^2\zeta}{2c_{\rm rel}\hbar\eta}E$$

Mass drops out!

all types of elementary particles contribute?

e *μ* *τ*

(*u* *d* *c* *s* *t* *b*) *3

W +/- ...

$$\varepsilon_0 = \frac{\zeta}{2c_{\rm rel}\hbar\eta}\left(\sum_j^{\rm e.\,p.} q_j^2\right)$$

$$P_0 = \frac{d}{V} = \frac{e^2\zeta}{2c_{\mathrm{rel}}\hbar\eta}E$$

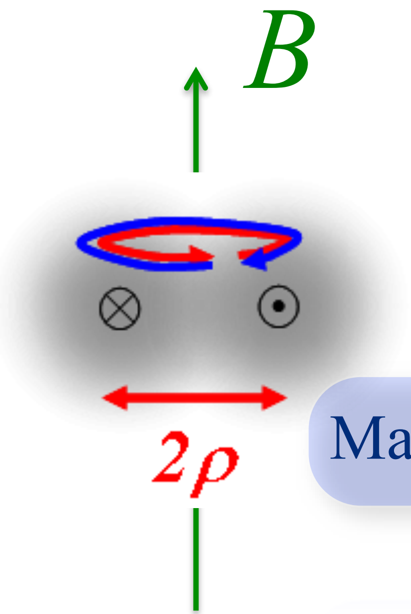
Mass drops out!

all types o
partic **?**

determine number of elementary particles
(discovered and undiscovered)

e μ τ
(*u d c s t b*) *3
W $^{+/-}$ ...

$$\varepsilon_0 = \frac{\zeta}{2c_{\mathrm{rel}}\hbar\eta}\left(\sum_j^{\mathrm{e.\,p.}} q_j^2\right)$$



$$d_{\mathrm{magn}} = 2iA = 2(e\nu)(\pi\varrho^2) = \frac{e^2}{m}\varrho^2 B$$

$$\varrho^2 = \xi\left(\frac{\hbar}{m_j c_{\mathrm{rel}}}\right)^2$$

Mass drops out!

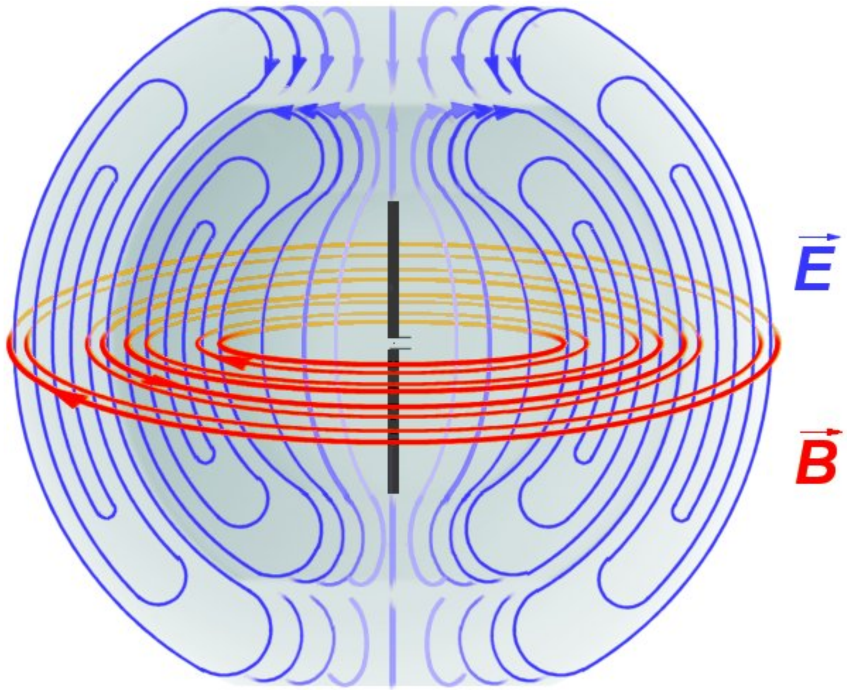
$$M_0 = \frac{\xi c_{\mathrm{rel}} e^2}{\eta\hbar}$$

$$\frac{1}{\mu_0} = \frac{\xi c_{\mathrm{rel}}}{\eta\hbar}\left(\sum_j^{\mathrm{e.\,p.}} q_j^2\right)$$

Vacuum impedance determines the power radiated by a dipole

$$P = e^2\omega^2 \left(\frac{d}{\lambda}\right)^2 \frac{\pi}{3} Z_0$$

$$Z_0 = 376.7 \text{ Ohms}$$



$$c_{\text{light}} = 1/\sqrt{\varepsilon_0 \mu_0} \quad \Rightarrow \quad c_{\text{light}} = c_{\text{rel}} \sqrt{\frac{2\xi}{\zeta}}$$

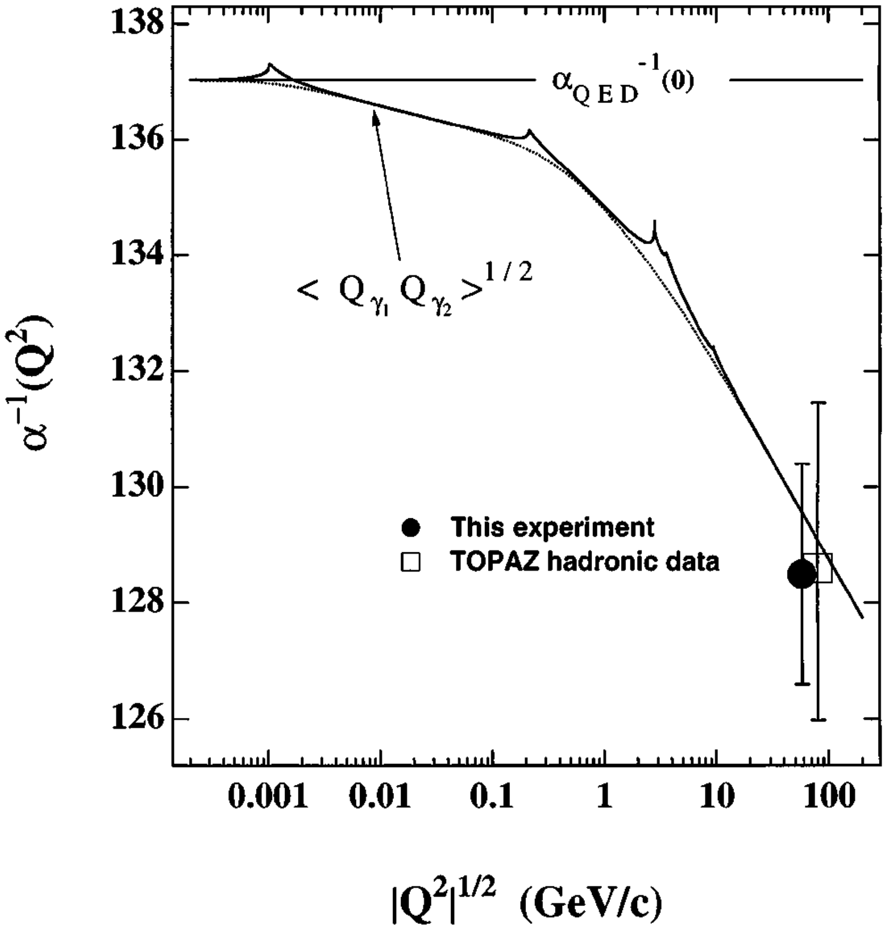
$$\frac{\zeta}{\xi} = 2$$

$$Z_0 = \frac{2\eta\hbar}{\zeta} \left(\sum_j^{\text{e. p.}} q_j^2 \right)^{-1} = 8218[\Omega] \frac{\eta}{\zeta} \left(\sum_j^{\text{e. p.}} \frac{q_j^2}{e^2} \right)^{-1}$$

$$\sum_j^{\text{e. p.}} \frac{q_j^2}{e^2} = \frac{2\hbar}{e^2 Z_0 \zeta} = 21.82 \frac{\eta}{\zeta}$$

$$\sum_j^{\text{known}} \frac{q_j^2}{e^2} \simeq 10$$

$$\eta \simeq 0.48 - 2.06$$



$$\alpha = \frac{e^2}{4\pi c\hbar\epsilon_0}$$

$$\alpha_0^{-1} = 137.04 = \text{constant} \sum_j^{\text{e. p.}} \frac{q_j^2}{e^2}$$

$$\alpha_{58\,\text{GeV}}^{-1} = 128.5 \pm 2.5$$

$$= \text{constant} \sum_j^{\text{e. p.}>58\,\text{GeV}} \frac{q_j^2}{e^2}$$

$$\frac{\alpha_0^{-1}}{\alpha_{58\,\mathrm{GeV}}^{-1}} = \frac{\sum_{\mathrm{all}}}{\sum_{>58\,\mathrm{GeV}}} = \frac{137.04}{128.05 \pm 2.5}$$

$$\sum_{\mathrm{all}} = 104 \begin{cases} +43 \\ -24 \end{cases}$$

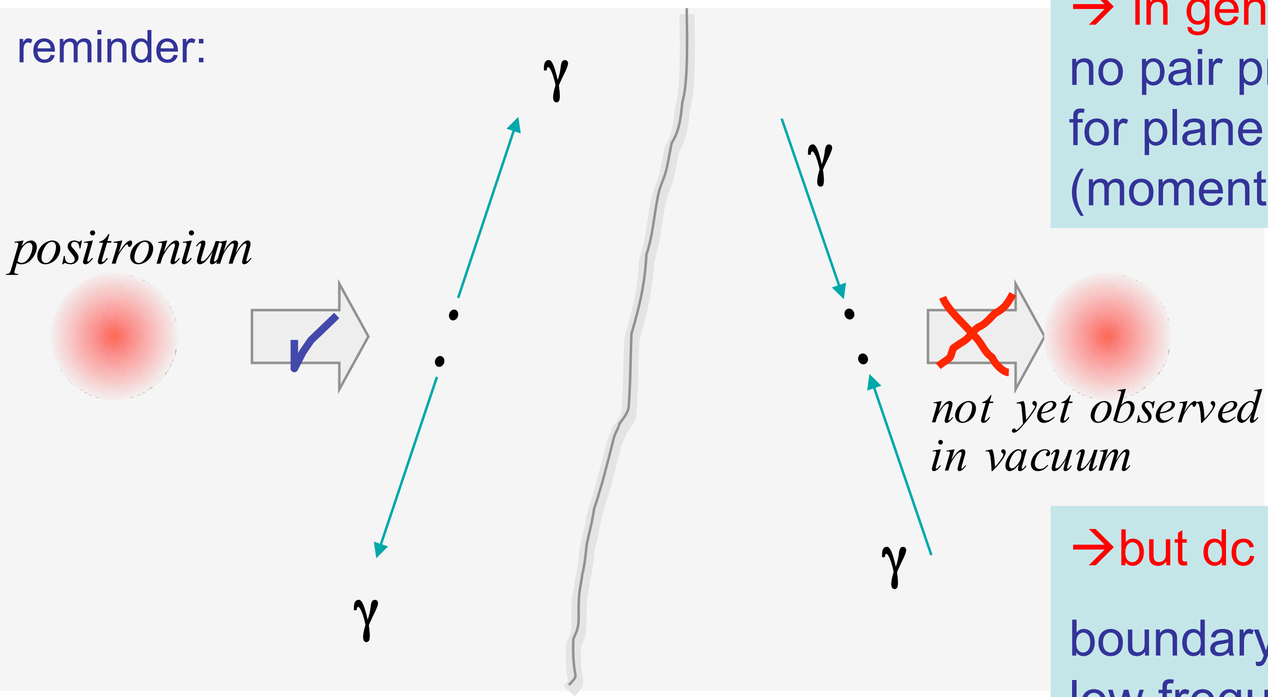
J. Schwinger,
Phys. Rev. 82, 664 (1951)

criticism of this simple model

- model not Lorentz invariant

Lorentz invariant quantities: $\vec{E}^2 - \vec{H}^2$, $\vec{E} \cdot \vec{H}$
are both zero for a plane wave
→ no pair production

reminder:



→ in general
no pair production possible
for plane wave
(momentum conservation)

→but dc field okay

boundary between dc and
low frequency?

Conclusions

• Our simple model predicts a finite number of charged elementary particles and that it relates this number to low-energy properties of the electromagnetic field

• The value predicted by the model is determined by the relative properties of the electric and magnetic interaction of light with the quantum vacuum and is independent of the number of elementary particles.

• We have shown an intimate relationship between the properties of the quantum vacuum and the constants in Maxwell's equations.