INTERNATIONAL WORKSHOP ON QUANTUM COHERENCE & DECOHERENCE
IWQCD1

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FAPESP

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INCT-IQ

Shielding quantum discord through continuous dynamical decoupling

F. F. Fanchini,
Emanuel F. de Lima, Leonardo K. Castelano

Outline

- ***Protecting Quantum Information***
  - *QECC*
  - *Decoherence Free Subspace*
  - *Dynamical Decoupling*

- ***Dynamical Decoupling***
  - *Magnus Expansion*
  - *Time Scales*
  - *Experimental Realizations*

- ***Continuous Dynamical Decoupling***
  - *Protecting quantum information against one class of error*
  - *Protecting quantum information against general errors*
  - *Protecting Entanglement*

- ***Quantum Discord***
  - *Why to study quantum discord*
  - *Peculiarities involving quantum discord*

- ***Shielding Quantum Discord throught Continuous DD***

Quantum Error Correction Code

Peter Shor

Andrew Steane

Shor PW
Scheme for reducing decoherence in quantum computer memory
PRA 52 R2493 OUT 1995

Steane AM
Error correcting codes in quantum theory
PRL 77, 793 JUL 1996

Quantum Error Correction Code - example

$$\begin{aligned} |\uparrow\uparrow\uparrow\rangle &= |0\rangle_L \\ |\downarrow\downarrow\downarrow\rangle &= |1\rangle_L \end{aligned}$$

$$\alpha|0\rangle_L + \beta|1\rangle_L = \alpha|\uparrow\uparrow\uparrow\rangle + \beta|\downarrow\downarrow\downarrow\rangle$$

Suppose an bit flip error on qubit 2

$$\alpha|\uparrow\downarrow\uparrow\rangle + \beta|\downarrow\uparrow\downarrow\rangle$$

Decoherence Free Subspace

Paolo Zanardi

Mario Rasetti

Zanardi P, Rasetti M
Noiseless quantum codes
PRL 79, 3306 OUT 1997

Decoherence Free Subspace - example

Suppose a collective dephasing error

$$H_{int} = \left(\sigma_z^{(1)} + \sigma_z^{(2)}\right) \otimes B$$

Is it possible to codify logical qubits in a DFS ???

$$\begin{aligned} |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle &= |0\rangle_L \\ |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle &= |1\rangle_L \end{aligned}$$

Dynamical Decoupling

Lorenza Viola

Seth Lloyd

Viola L, Lloyd S
Dynamical suppression of decoherence in two-state quantum systems
PRA 58, 2733 OUT 1998

Dynamical Decoupling

Magnus Expansion

Suppose that we want a solution to the equation given by:

$$\frac{dU(t)}{dt} = H(t)U(t)$$

Magnus proposed an exponential solution

$$U(t) = exp[A(t)]$$

where A(t) is a linear operator to be determined

$$\frac{dU(t)}{dt} = H(t)U(t)$$

$$U(t) = exp[A(t)]$$

The Magnus idea was to write

$$U(t)A(t)U^{\dagger}(t) = A(t)$$

and then calculate dA(t)/dt

So, to solve dA(t)/dt we expand A(t) in time

$$\frac{d}{dt}(A_1(t) + A_2(t) + A_3(t) + ...) \quad =$$

$$H(t) - \frac{1}{2}[A(t), H(t)] + \frac{1}{12}[A(t), [A(t), H(t)]] + ...$$

Solving,

$$\boxed{A_1(t) = \int_0^t H(t')dt'}$$

The main point of DD is to determine a H(t) that results in $A_1(t)=0$.

$$A_2(t) = \frac{1}{2}\int_0^t [H(t'), A_1(t']dt'.$$

$$A_3(t) = \frac{1}{2}\int_0^t [H(t'), A_2(t']dt' + \frac{1}{12}\int_0^t [[H_1(t'), A_1(t')], H(t')]dt'.$$

Magnus Expansion 1ª order

$$A_1(t) = \int_0^t H(t')dt'$$

Dynamical Decoupling

$$H(t) = H_c(t) + H_E + H_{int}$$

In the interaction Picture

$$\tilde{H}(t) = H_E + U_c^\dagger(t) H_{int} U_c(t)$$

General Prescription of Dynamical Decoupling

$$\int_0^{t_c} dt U_c^\dagger(t) H_{int} U_c(t) = 0$$

Time Scales

$$\tau << T_2 < T_1$$

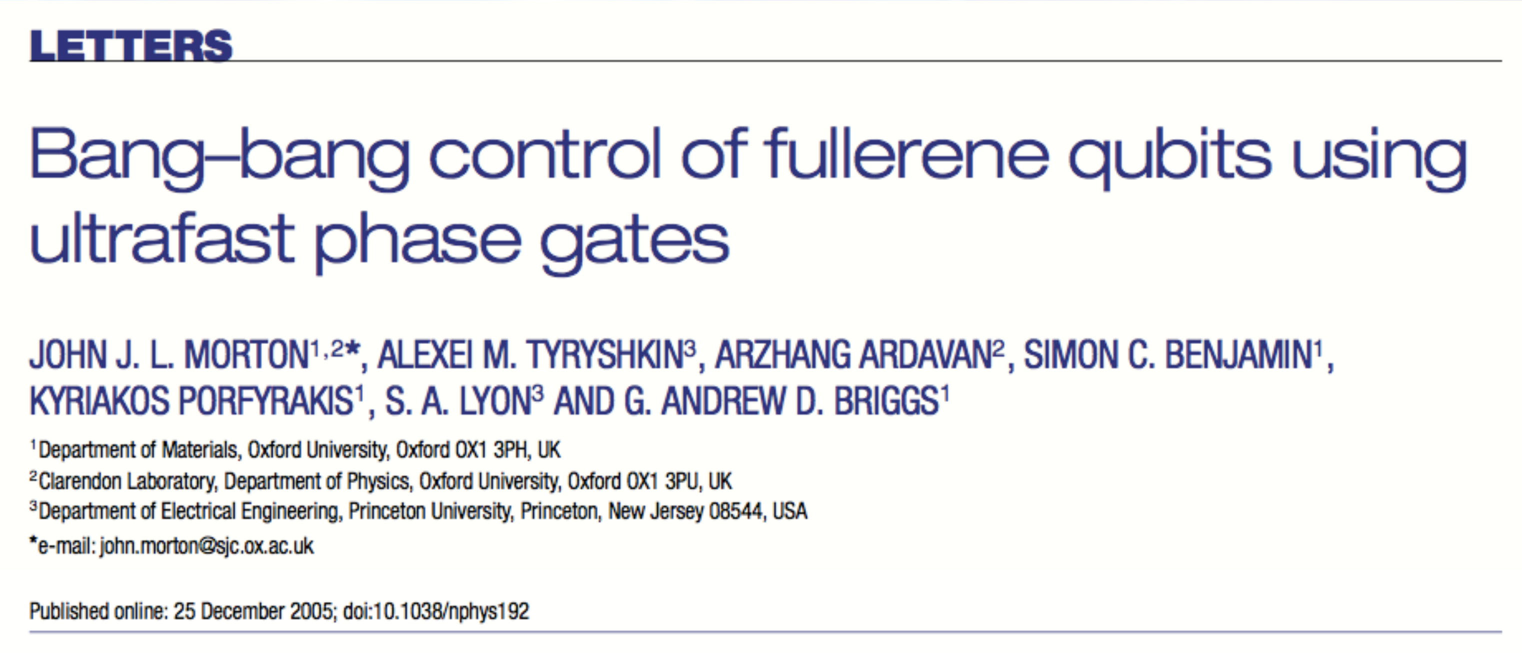
$\tau \longrightarrow$ reservoir correlation time $\left(= \frac{1}{cutoff} \right)$

$T_2 \longrightarrow$ decoherence time (without energy exchange)

$T_1 \longrightarrow$ dissipation time (with energy exchange)

$$\underline{t_{op}}$$

Time of a quantum gate operation



LETTERS

Bang–bang control of fullerene qubits using ultrafast phase gates

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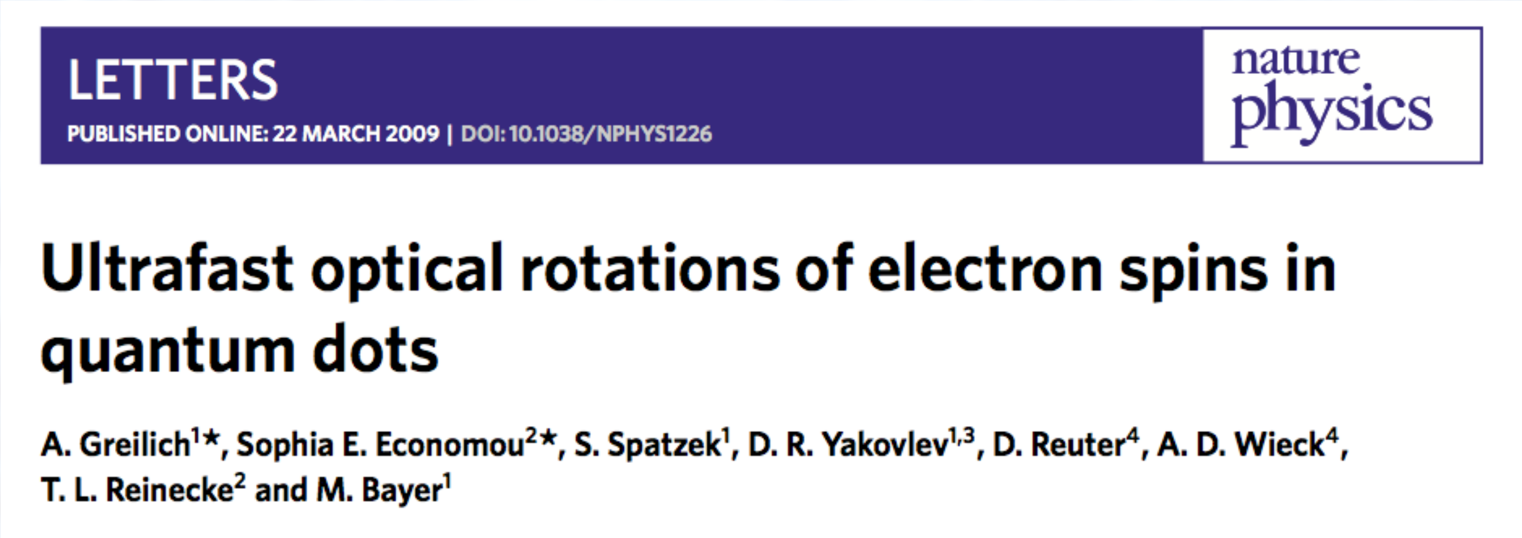
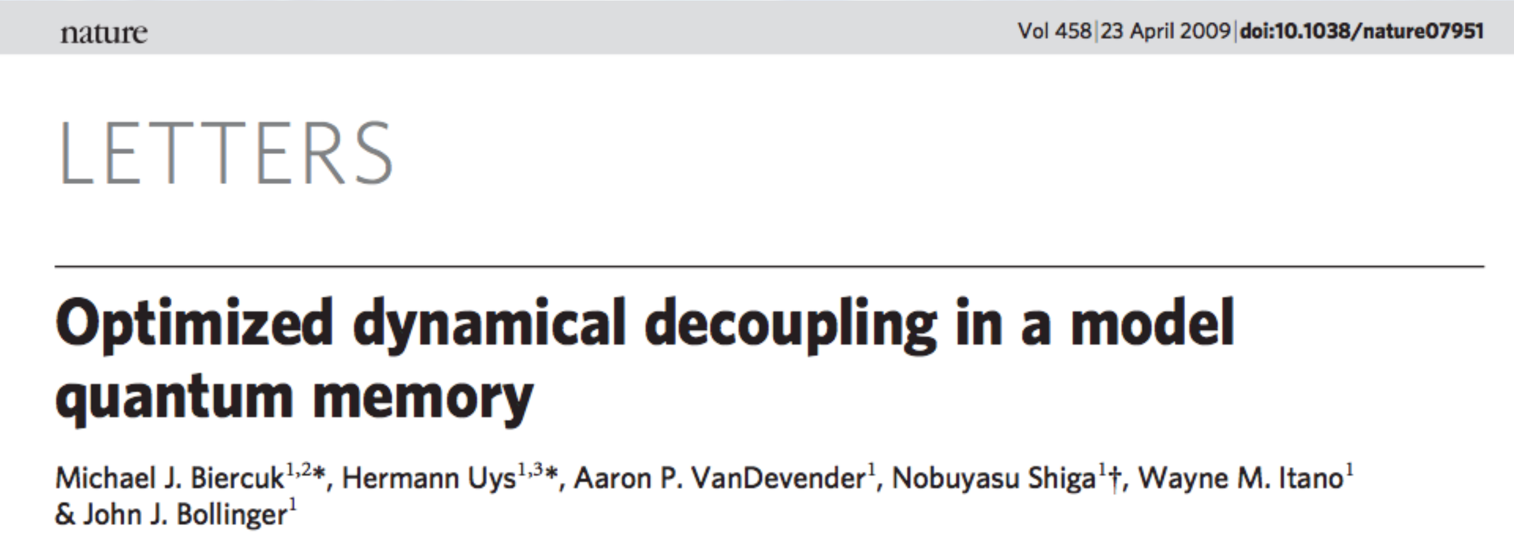
Published online: 25 December 2005; doi:10.1038/nphys192



LETTERS
PUBLISHED ONLINE: 22 MARCH 2009 | DOI: 10.1038/NPHYS1226
nature physics

Ultrafast optical rotations of electron spins in quantum dots

A. Greilich[1]*, Sophia E. Economou[2]*, S. Spatzek[1], D. R. Yakovlev[1,3], D. Reuter[4], A. D. Wieck[4], T. L. Reinecke[2] and M. Bayer[1]



nature Vol 458|23 April 2009|doi:10.1038/nature07951

LETTERS

Optimized dynamical decoupling in a model quantum memory

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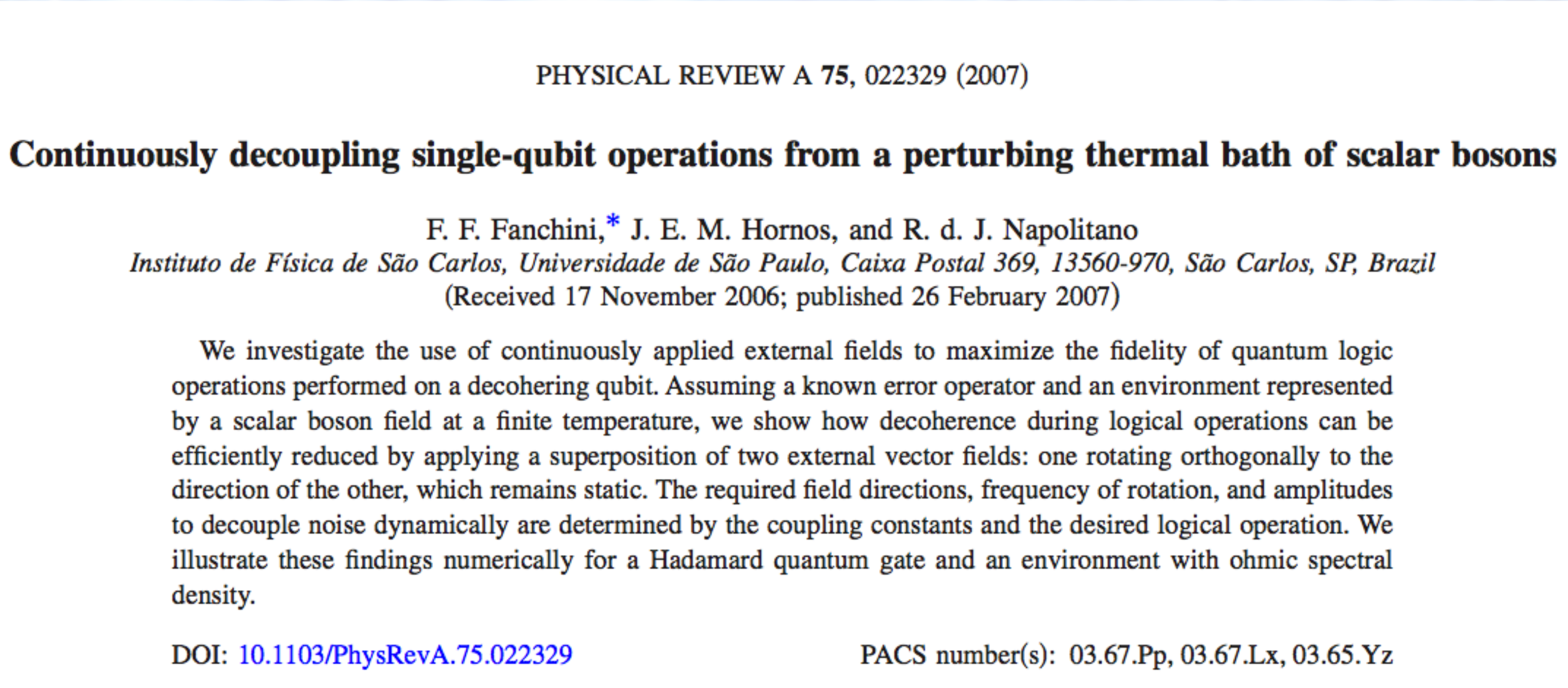
[redacted]

CONTINUOUS DYNAMICAL DECOUPLING



PHYSICAL REVIEW A **75**, 022329 (2007)

Continuously decoupling single-qubit operations from a perturbing thermal bath of scalar bosons

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(Received 17 November 2006; published 26 February 2007)

We investigate the use of continuously applied external fields to maximize the fidelity of quantum logic operations performed on a decohering qubit. Assuming a known error operator and an environment represented by a scalar boson field at a finite temperature, we show how decoherence during logical operations can be efficiently reduced by applying a superposition of two external vector fields: one rotating orthogonally to the direction of the other, which remains static. The required field directions, frequency of rotation, and amplitudes to decouple noise dynamically are determined by the coupling constants and the desired logical operation. We illustrate these findings numerically for a Hadamard quantum gate and an environment with ohmic spectral density.

DOI: 10.1103/PhysRevA.75.022329 PACS number(s): 03.67.Pp, 03.67.Lx, 03.65.Yz

We begin our studies considering a thermal bath of scalar bosons:

$$H_{\text{int}} = (\lambda \cdot \sigma)B + (\lambda^{*} \cdot \sigma)B^{\dagger}$$

Our objective is determine an unitary transformation that satisfy the general prescription of dynamical decoupling

$$\int_{0}^{t_c} dt U_c^{\dagger}(t) H_{int} U_c(t) = 0$$

Any time-dependent unitary transformation of a qubit can always be expressed as

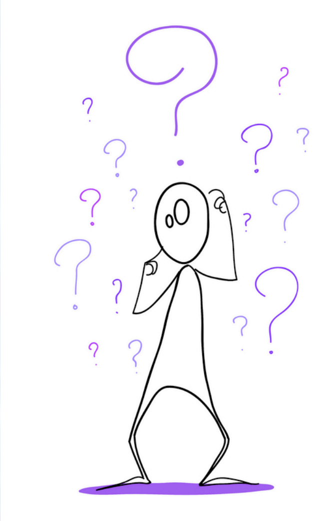
$$U(t) = I\cos[\alpha(t)] - i\sigma \cdot \hat{\mathbf{u}}(t)\sin[\alpha(t)]$$

Protecting a Hadamard Quantum Gate

To apply a protected quantum gate we write the unitary evolution as:

$$\boxed{U(t)=U_c(t)U_0(t)}$$

and the Hamiltonian ???



$$\mathcal{U}_S(t) = \mathcal{U}_c(t)\mathcal{U}_0(t)$$

$$\begin{aligned} \mathcal{U}_c(t) &= I\cos(2n\pi t/\tau) - i\hat{\mathbf{u}}_c \cdot \boldsymbol{\sigma}\sin(2n\pi t/\tau) \\ \mathcal{U}_0(t) &= I\cos(\theta_0 t/\tau) - i\hat{\mathbf{u}}_0 \cdot \boldsymbol{\sigma}\sin(\theta_0 t/\tau) \end{aligned}$$

The Hamiltonian:

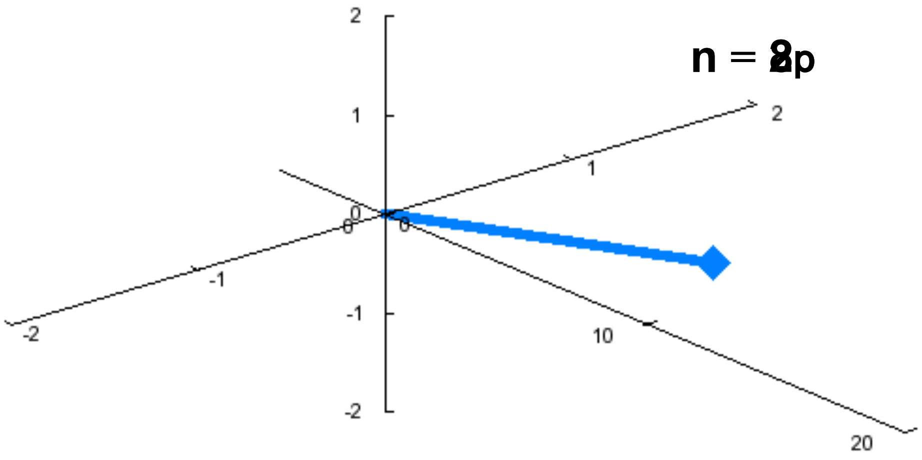
$$H_S(t) = \hbar\boldsymbol{\Omega}(t)\cdot\boldsymbol{\sigma} = i\hbar\frac{d\mathcal{U}_S(t)}{dt}\mathcal{U}_S^\dagger(t)$$

Quantum Gate + Protective Field

$$\left\{ \begin{aligned} \boldsymbol{\Omega}(t) &= [(2n\pi/\tau) + (\theta_0/\tau)\hat{\mathbf{u}}_c\cdot\hat{\mathbf{u}}_0]\hat{\mathbf{u}}_c \\ &\quad +(\theta_0/\tau)[\hat{\mathbf{u}}_c\times(\hat{\mathbf{u}}_0\times\hat{\mathbf{u}}_c)]\cos(2n\pi t/\tau) \\ &\quad +(\theta_0/\tau)(\hat{\mathbf{u}}_c\times\hat{\mathbf{u}}_0)\sin(2n\pi t/\tau) \end{aligned} \right.$$

Quantum Gate + Protective Field

S



$'\tau)$

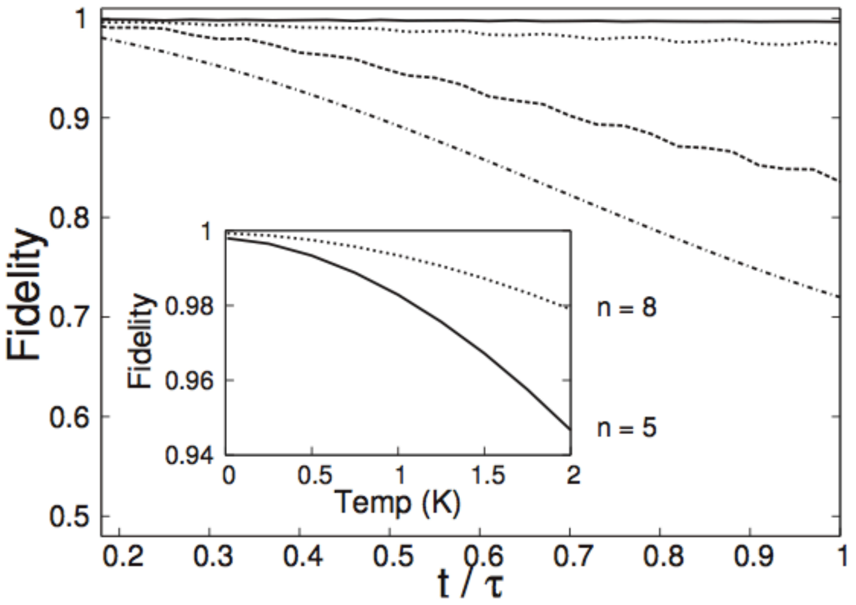


FIG. 1. Numerical solutions for the strategy example described in the text to overcome decoherence and dissipation during a Hadamard operation. The dot-dashed line represents the fidelity when $H_c=0$, while the solid line is the result for $n=5$ in Eq. (18) and $\hat{\mathbf{u}}_c$, chosen orthogonally to the error vector $\boldsymbol{\lambda}$, as in the text. The dotted and dashed lines represent the $n=5$ results when $\hat{\mathbf{u}}_c$ is tilted 10° and 30° toward the x axis, respectively. In the inset we show the fidelities at $t=\tau$ as functions of temperature and n.

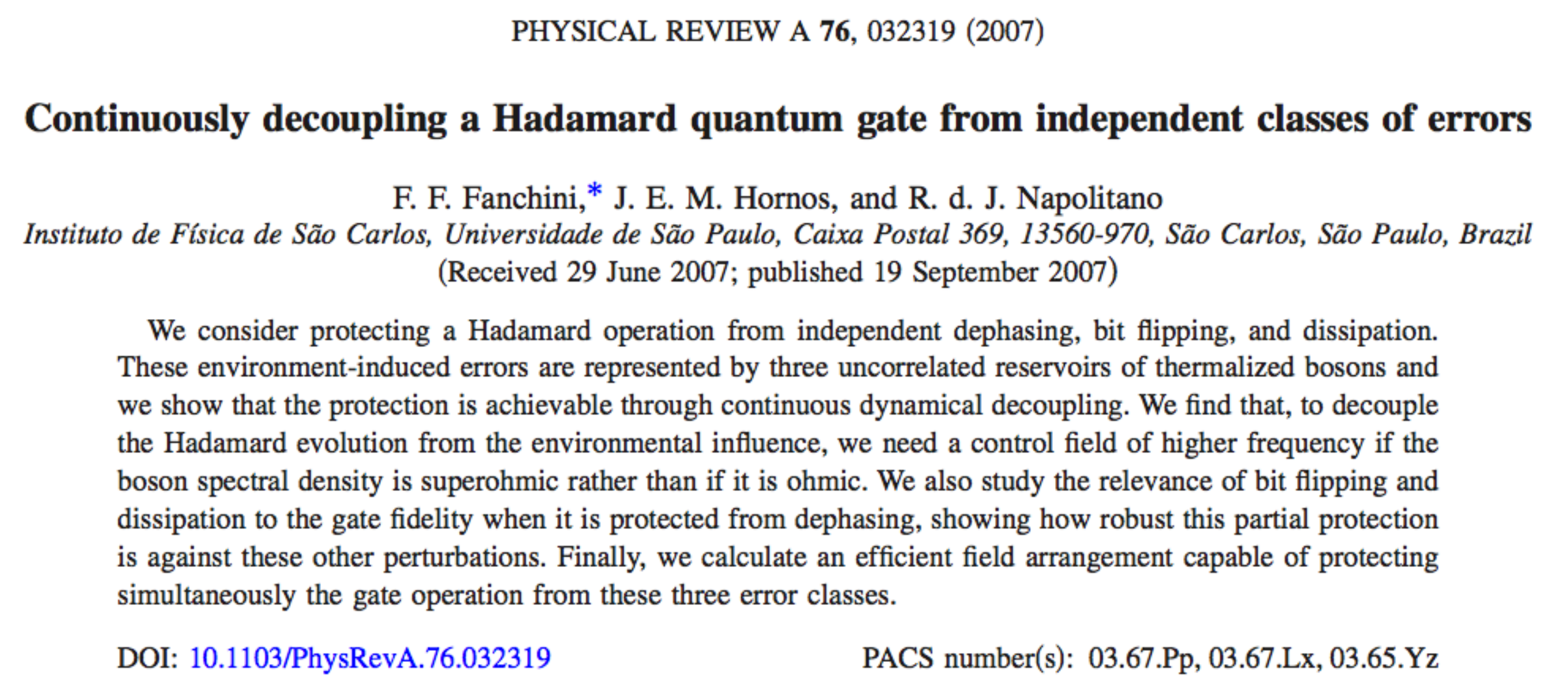
How to include the protection for general errors ?

One class of error

$$H_I = (\boldsymbol{\lambda}_k \cdot \boldsymbol{\sigma}) B_k + (\boldsymbol{\lambda}_k^* \cdot \boldsymbol{\sigma}) B_k^\dagger$$

Independent classes of errors

How to protect the quantum information in this situation ?



PHYSICAL REVIEW A **76**, 032319 (2007)

Continuously decoupling a Hadamard quantum gate from independent classes of errors

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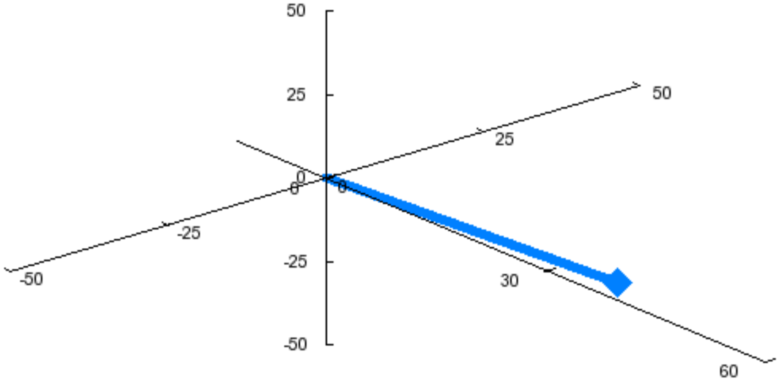
(Received 29 June 2007; published 19 September 2007)

We consider protecting a Hadamard operation from independent dephasing, bit flipping, and dissipation. These environment-induced errors are represented by three uncorrelated reservoirs of thermalized bosons and we show that the protection is achievable through continuous dynamical decoupling. We find that, to decouple the Hadamard evolution from the environmental influence, we need a control field of higher frequency if the boson spectral density is superohmic rather than if it is ohmic. We also study the relevance of bit flipping and dissipation to the gate fidelity when it is protected from dephasing, showing how robust this partial protection is against these other perturbations. Finally, we calculate an efficient field arrangement capable of protecting simultaneously the gate operation from these three error classes.

DOI: 10.1103/PhysRevA.76.032319 PACS number(s): 03.67.Pp, 03.67.Lx, 03.65.Yz

$$\mathbf{\Omega}(t) \quad = \quad (2n\pi/\tau)\hat{\mathbf{x}} + (2m\pi/\tau)[\cos(2n\pi/\tau)\hat{\mathbf{z}} - \sin(2n\pi/\tau)\hat{\mathbf{y}}]$$



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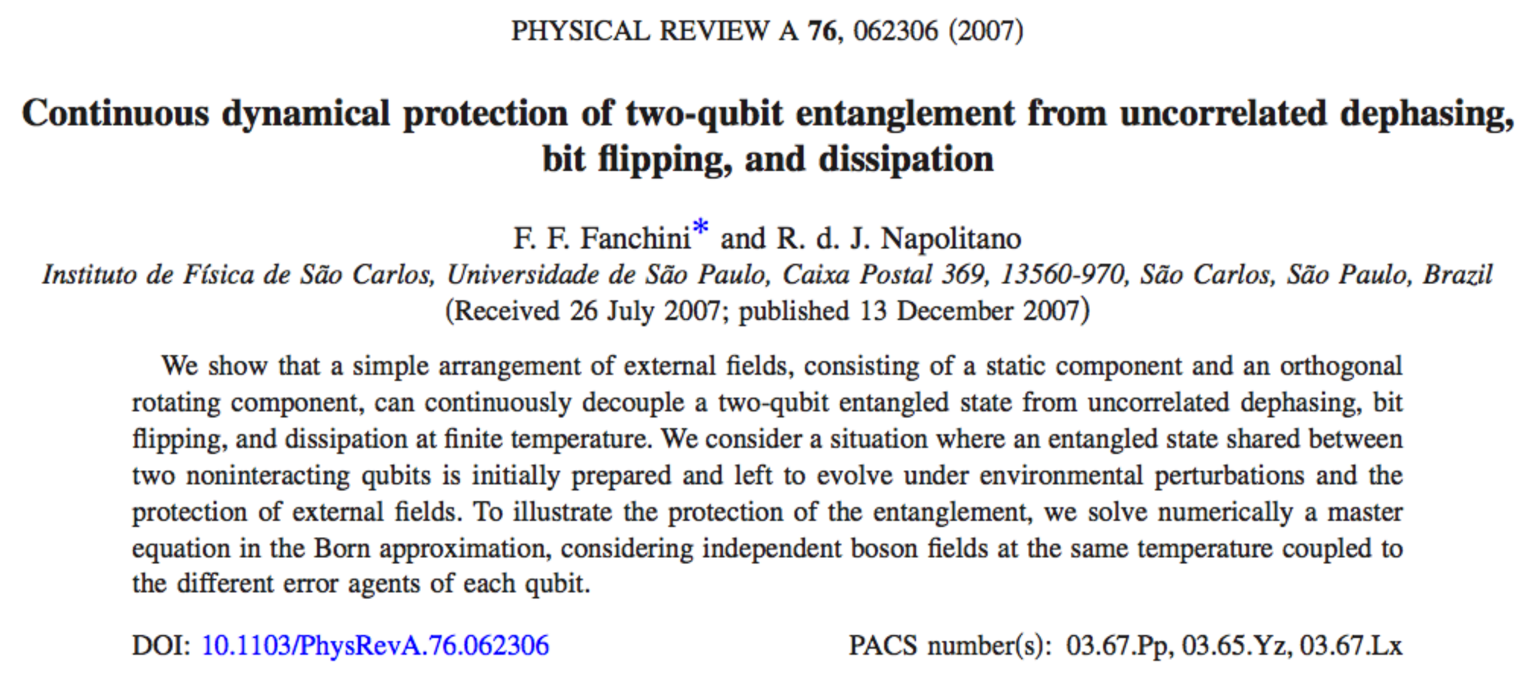

PHYSICAL REVIEW A **76**, 062306 (2007)

Continuous dynamical protection of two-qubit entanglement from uncorrelated dephasing, bit flipping, and dissipation

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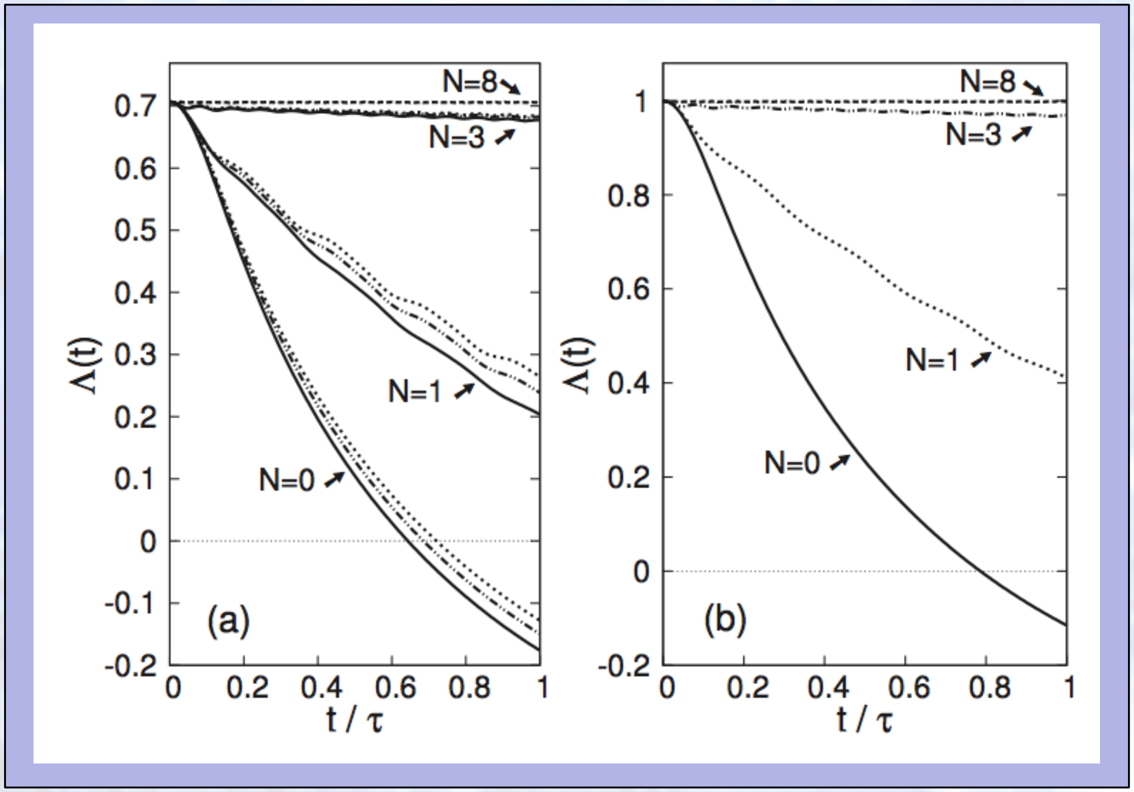
(Received 26 July 2007; published 13 December 2007)

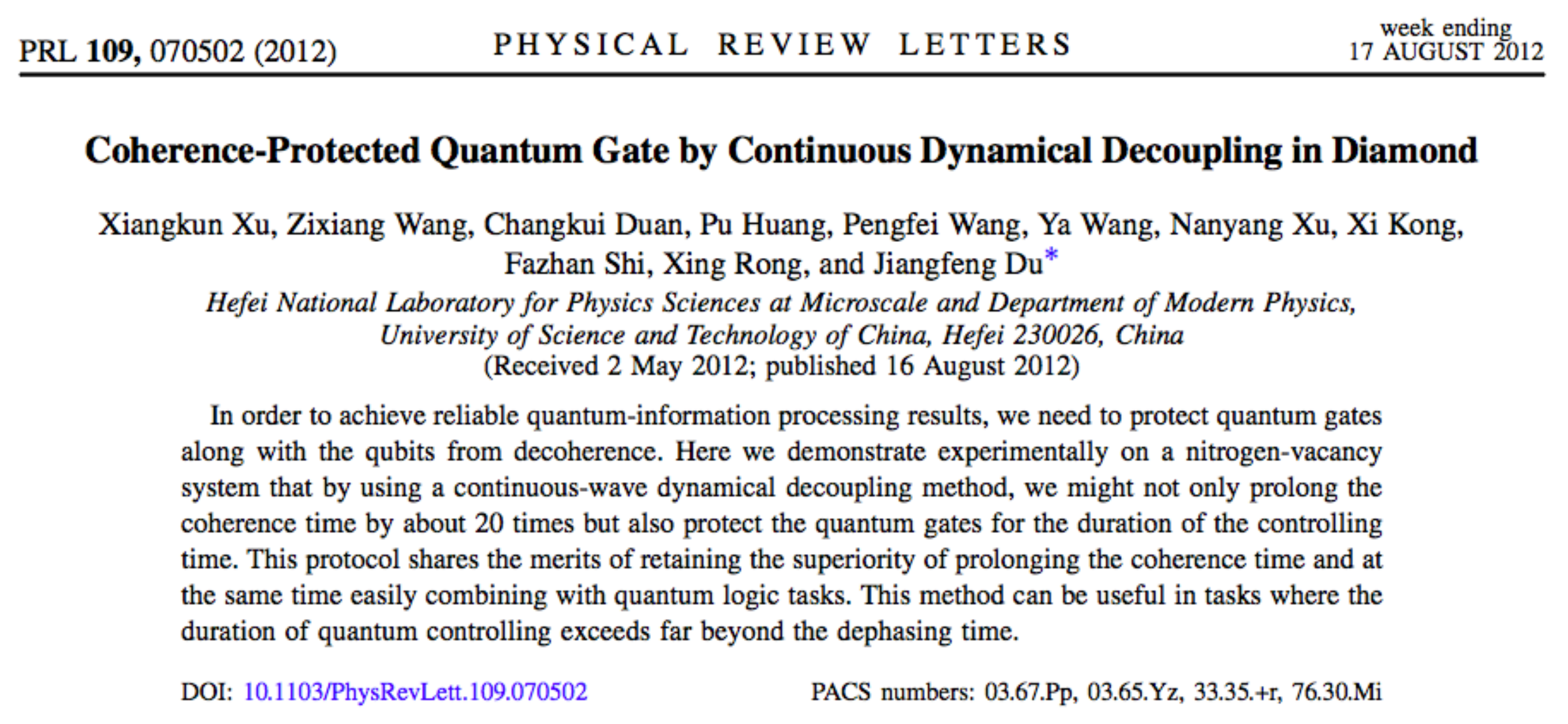
We show that a simple arrangement of external fields, consisting of a static component and an orthogonal rotating component, can continuously decouple a two-qubit entangled state from uncorrelated dephasing, bit flipping, and dissipation at finite temperature. We consider a situation where an entangled state shared between two noninteracting qubits is initially prepared and left to evolve under environmental perturbations and the protection of external fields. To illustrate the protection of the entanglement, we solve numerically a master equation in the Born approximation, considering independent boson fields at the same temperature coupled to the different error agents of each qubit.

DOI: 10.1103/PhysRevA.76.062306 PACS number(s): 03.67.Pp, 03.65.Yz, 03.67.Lx

$$H_c(t) = \boldsymbol{\Omega}(t)\cdot(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)$$

$$\boldsymbol{\Omega}(t) = \hat{\mathbf{x}}n_x\omega + n_z\omega[\hat{\mathbf{z}}\cos(n_x\omega t) - \hat{\mathbf{y}}\sin(n_x\omega t)]$$





PRL **109,** 070502 (2012) PHYSICAL REVIEW LETTERS week ending 17 AUGUST 2012

Coherence-Protected Quantum Gate by Continuous Dynamical Decoupling in Diamond

Xiangkun Xu, Zixiang Wang, Changkui Duan, Pu Huang, Pengfei Wang, Ya Wang, Nanyang Xu, Xi Kong,
Fazhan Shi, Xing Rong, and Jiangfeng Du*
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(Received 2 May 2012; published 16 August 2012)

In order to achieve reliable quantum-information processing results, we need to protect quantum gates along with the qubits from decoherence. Here we demonstrate experimentally on a nitrogen-vacancy system that by using a continuous-wave dynamical decoupling method, we might not only prolong the coherence time by about 20 times but also protect the quantum gates for the duration of the controlling time. This protocol shares the merits of retaining the superiority of prolonging the coherence time and at the same time easily combining with quantum logic tasks. This method can be useful in tasks where the duration of quantum controlling exceeds far beyond the dephasing time.

DOI: 10.1103/PhysRevLett.109.070502 PACS numbers: 03.67.Pp, 03.65.Yz, 33.35.+r, 76.30.Mi

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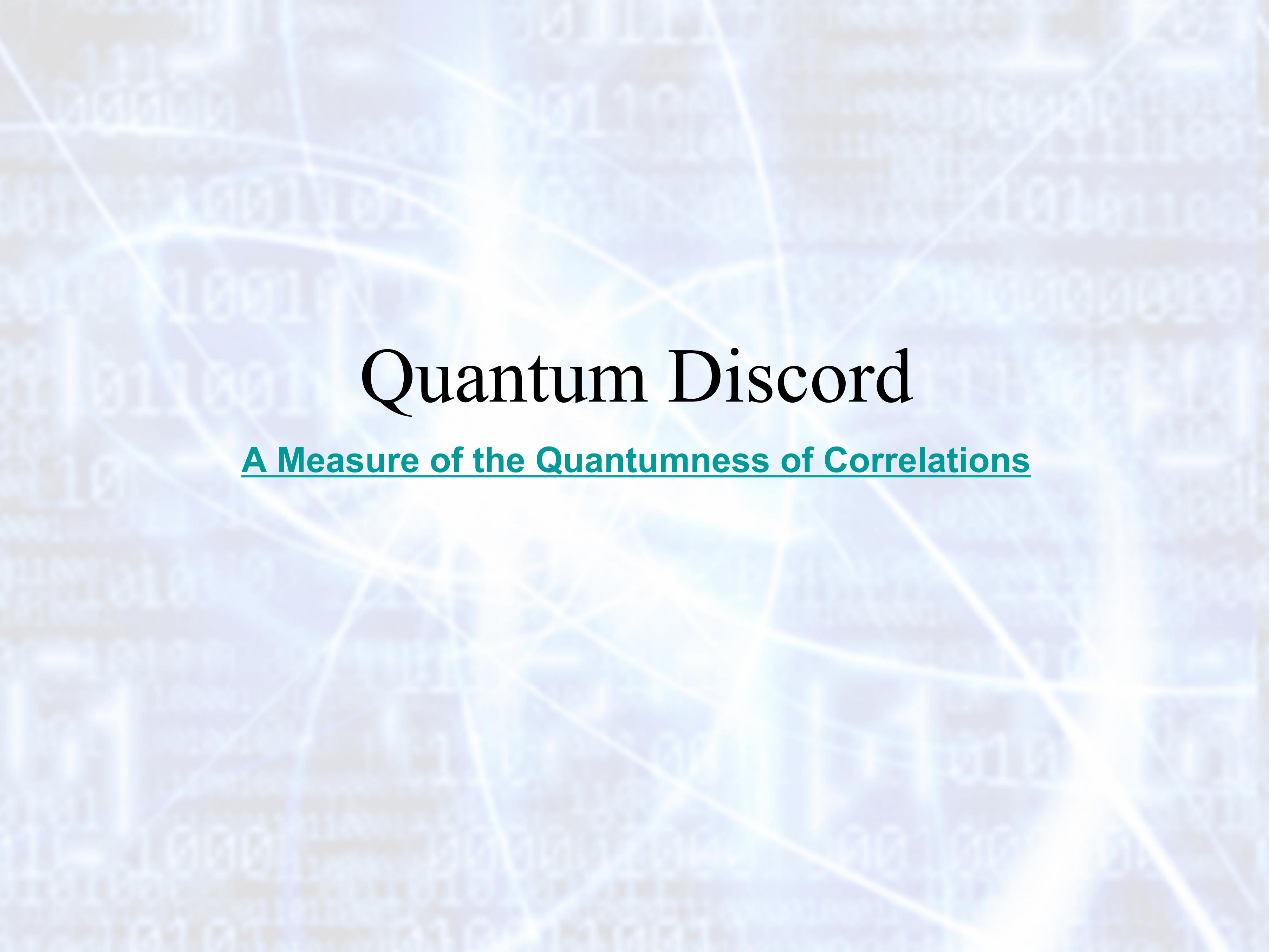


Quantum Discord

A Measure of the Quantumness of Correlations

Mutual Information

Classically the mutual information is defined as

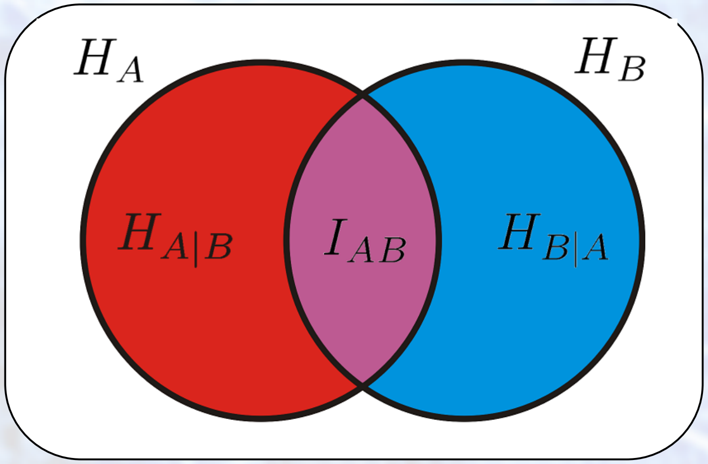
$$\boxed{I_{AB} = H_A + H_B - H_{AB}}$$

"mutual information measures
the information that
A and B share"

Or, by means of Bayes rule it can be written as

$$\boxed{I_{AB} = H_A - H_{A|B}}$$

"it measures how much
knowing one of the
subsystems reduces our
uncertainty about the other "



Venn Diagram

In the quantum world the mutual information is defines as

$$I_{AB} = S_A + S_B - S_{AB}$$

For quantum systems, generally the Mutual Information between A and B is <u>NOT</u> locally accessible.
There is an amount of <u>LOCAL INACESSIBLE INFORMATION</u> (LII)

Classical Correlation

Measure the maximum amount of the mutual information that is <u>locally accessible</u>

$$J^{\leftarrow}_{AB} = \max_{\{\Pi_k\}} \left[S_A - \sum_k p_k S_{A|k} \right]$$

L. Henderson and V. Vedral, J. Phys. A 34, 6899 (2001).

$$\begin{aligned} S_{A|k} &\equiv S(\rho_{A|k}) \\ \rho_{A|k} &= \mathrm{Tr}_B(\Pi_k \rho_{AB} \Pi_k)/\mathrm{Tr}_{AB}(\Pi_k \rho_{AB} \Pi_k) \end{aligned}$$

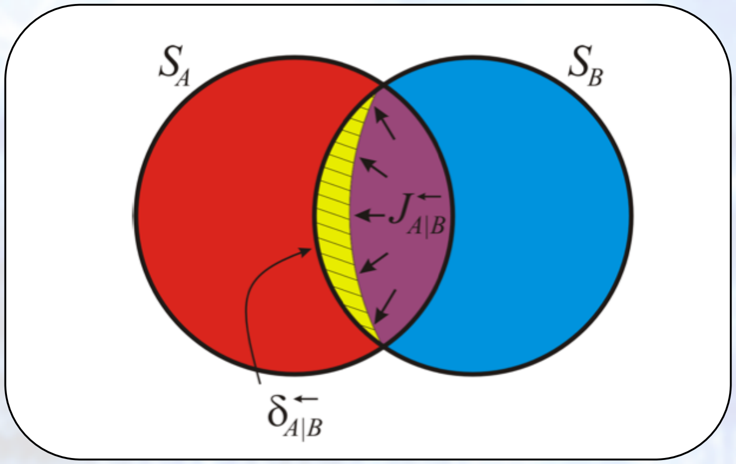
$\{\Pi_k\} \longrightarrow$ **POVM**

Quantum Discord

$$\delta^{\leftarrow}_{AB} = I_{AB} - J^{\leftarrow}_{AB}$$

H. Ollivier and W. H. Zurek, Phys. Rev. Lett. 88, 017901 (2002).

Measures the amount of the mutual information that is NOT Locally accessible



Why to study Quantum discord?

Reason 1. ***The deterministic quantum computation with one clean qubit.***

Reason 2. ***It is connected with fundamental questions of the quantum information theory***

- *Entanglement irreversibility*
- *Connected the negative signal of the conditional entropy*
- *Entanglement sudden death*
- *State Merging Protocol*
- *Linked with the entanglement in a measurement*
- *…*

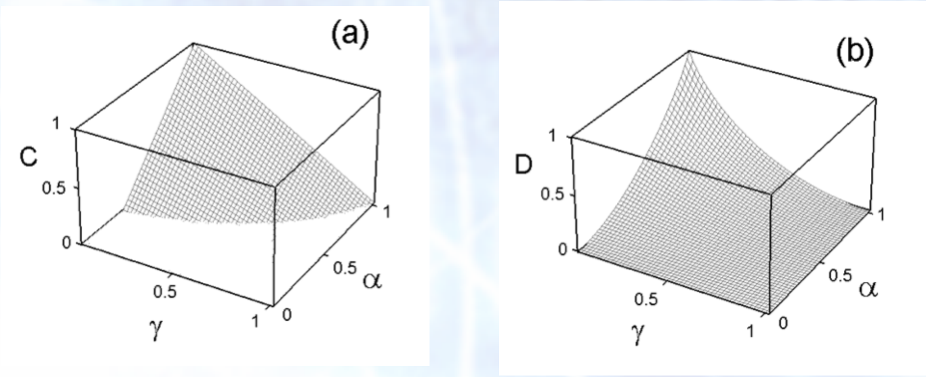
Reason 3. ***As an efficient detector of quantum phase transition***

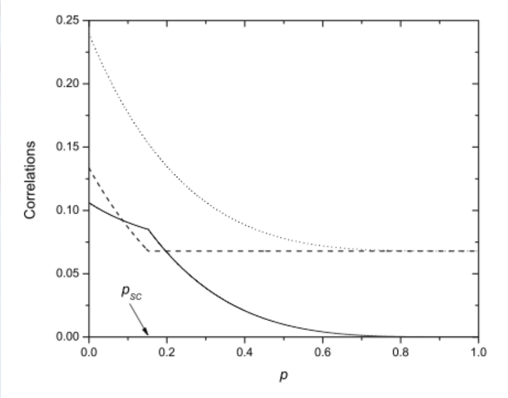
.
.
.

Peculiarities of the Quantum discord

- *Robustness to sudden death*

  Phys. Rev. 80, 024103 (2009)

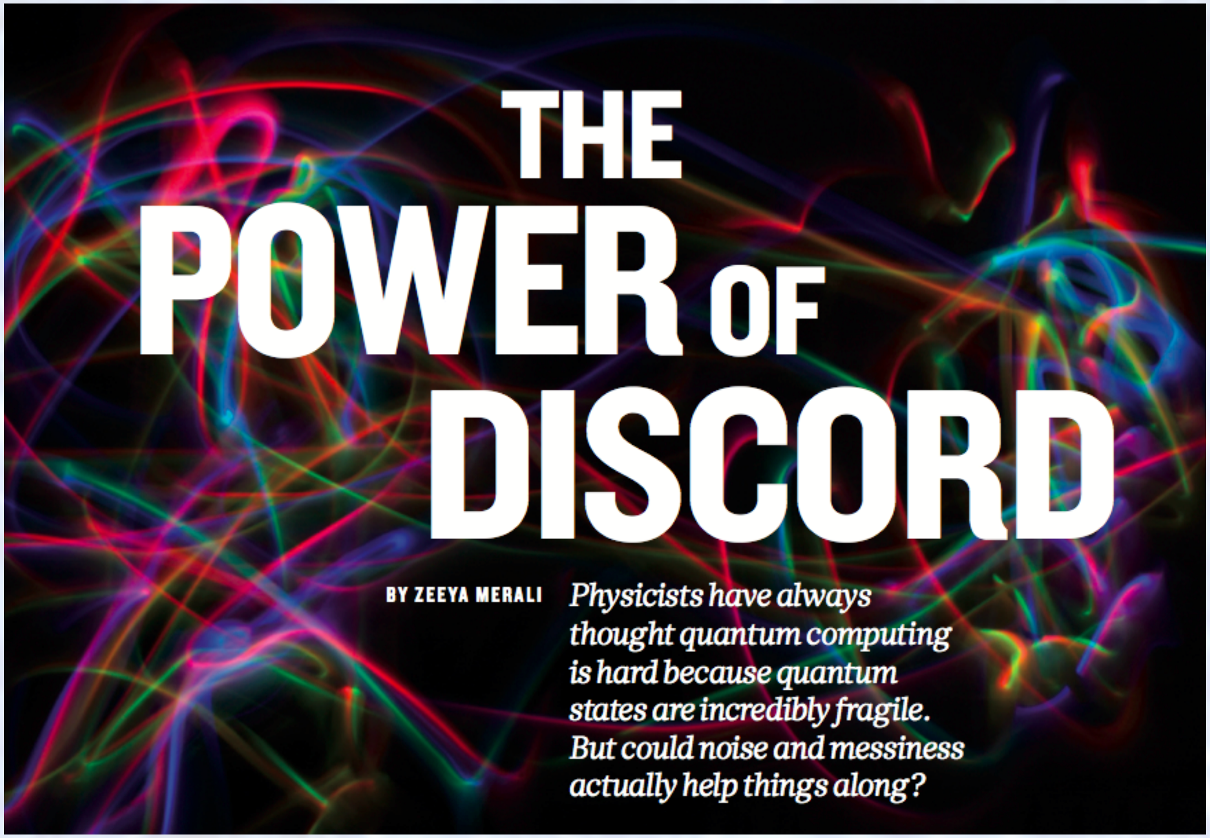




- *Sudden changes during the dissipative dynamics*

  Phys. Rev. A 80, 044102 (2009)

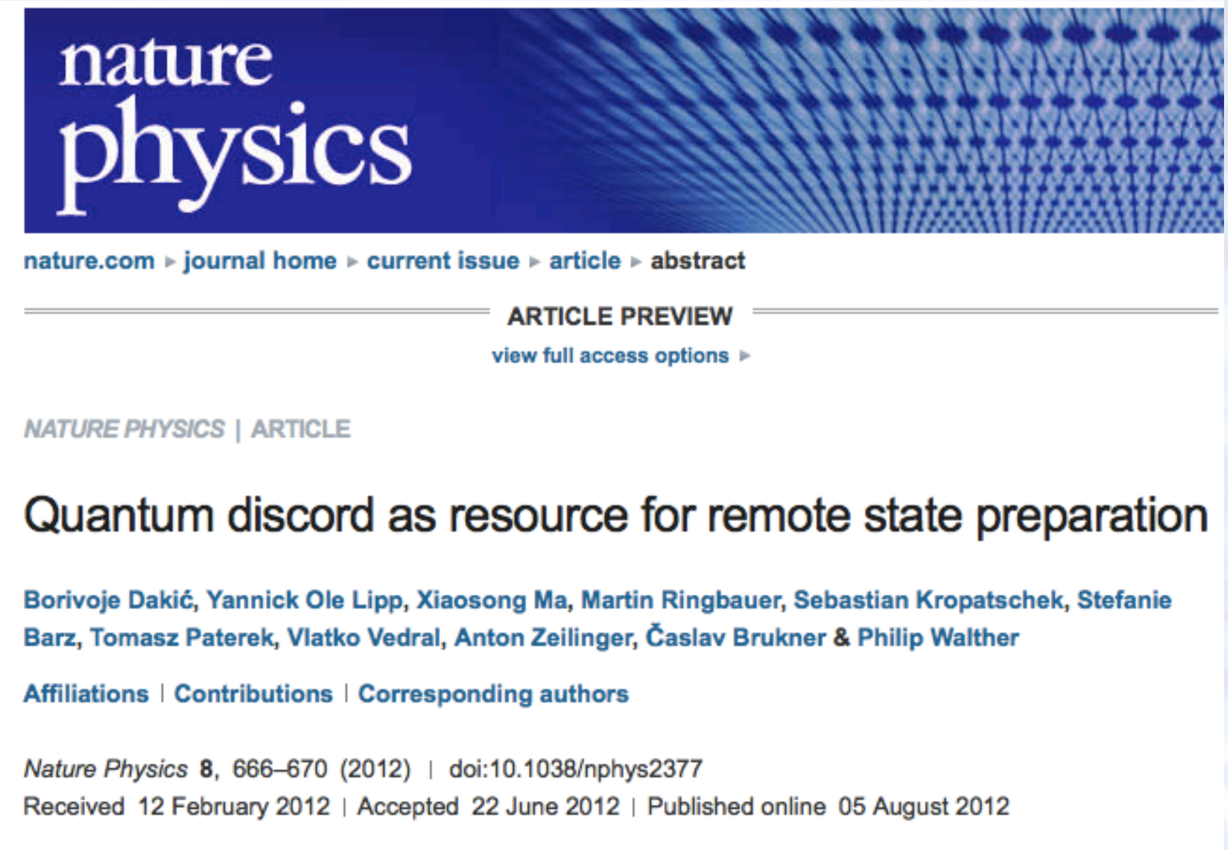
- *Assymetry* $\delta^{\leftarrow}_{AB} \neq \delta^{\leftarrow}_{BA}$



THE POWER OF DISCORD

BY ZEEYA MERALI

Physicists have always thought quantum computing is hard because quantum states are incredibly fragile. But could noise and messiness actually help things along?



nature physics

NATURE PHYSICS | ARTICLE

Quantum discord as resource for remote state preparation

Borivoje Dakić, Yannick Ole Lipp, Xiaosong Ma, Martin Ringbauer, Sebastian Kropatschek, Stefanie Barz, Tomasz Paterek, Vlatko Vedral, Anton Zeilinger, Časlav Brukner & Philip Walther

Nature Physics **8**, 666–670 (2012) | doi:10.1038/nphys2377
Received 12 February 2012 | Accepted 22 June 2012 | Published online 05 August 2012

nature physics

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ARTICLE PREVIEW

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NATURE PHYSICS | ARTICLE

Observing the operational significance of discord consumption

Mile Gu, Helen M. Chrzanowski, Syed M. Assad, Thomas Symul, Kavan Modi, Timothy C. Ralph, Vlatko Vedral & Ping Koy Lam

Affiliations | Contributions | Corresponding authors

Nature Physics **8**, 671–675 (2012) | doi:10.1038/nphys2376
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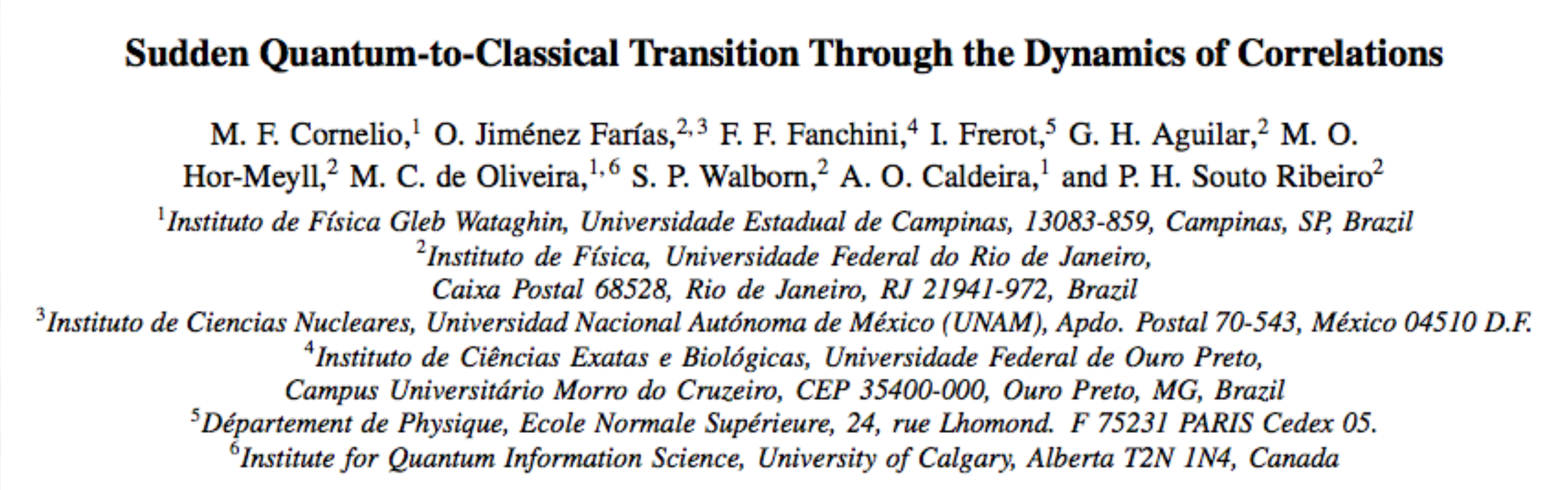

Sudden Quantum-to-Classical Transition Through the Dynamics of Correlations

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arXiv:1203.5068

Accepted in PRL

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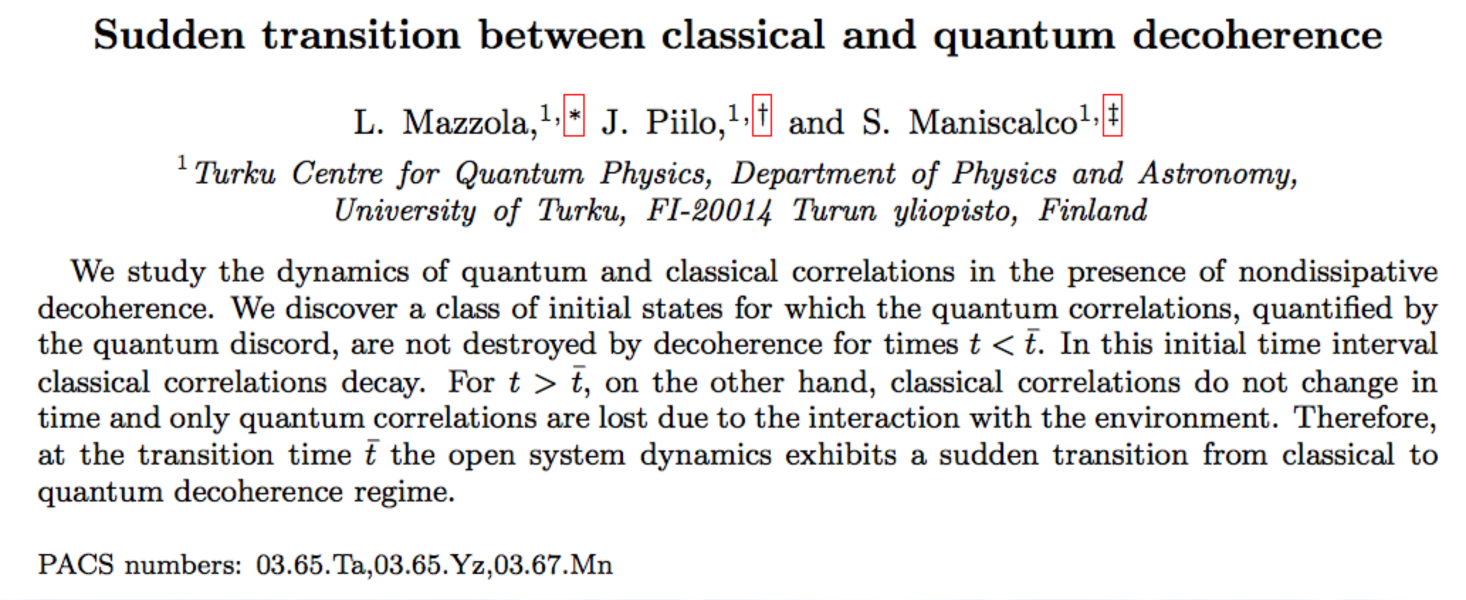

Sudden transition between classical and quantum decoherence

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We study the dynamics of quantum and classical correlations in the presence of nondissipative decoherence. We discover a class of initial states for which the quantum correlations, quantified by the quantum discord, are not destroyed by decoherence for times $t < \bar{t}$. In this initial time interval classical correlations decay. For $t > \bar{t}$, on the other hand, classical correlations do not change in time and only quantum correlations are lost due to the interaction with the environment. Therefore, at the transition time $\bar{t}$ the open system dynamics exhibits a sudden transition from classical to quantum decoherence regime.

PACS numbers: 03.65.Ta,03.65.Yz,03.67.Mn

Phys. Rev. Lett. 104, 200401 (2010)

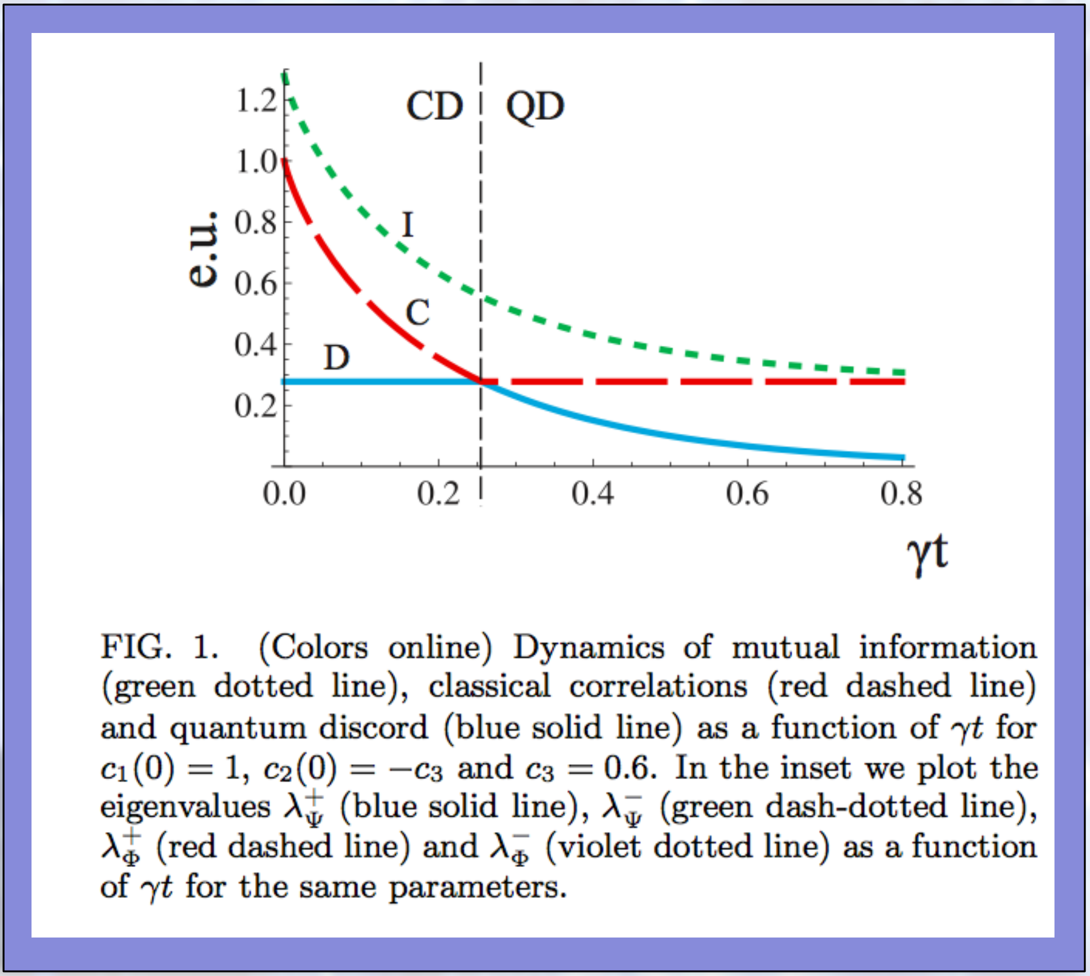
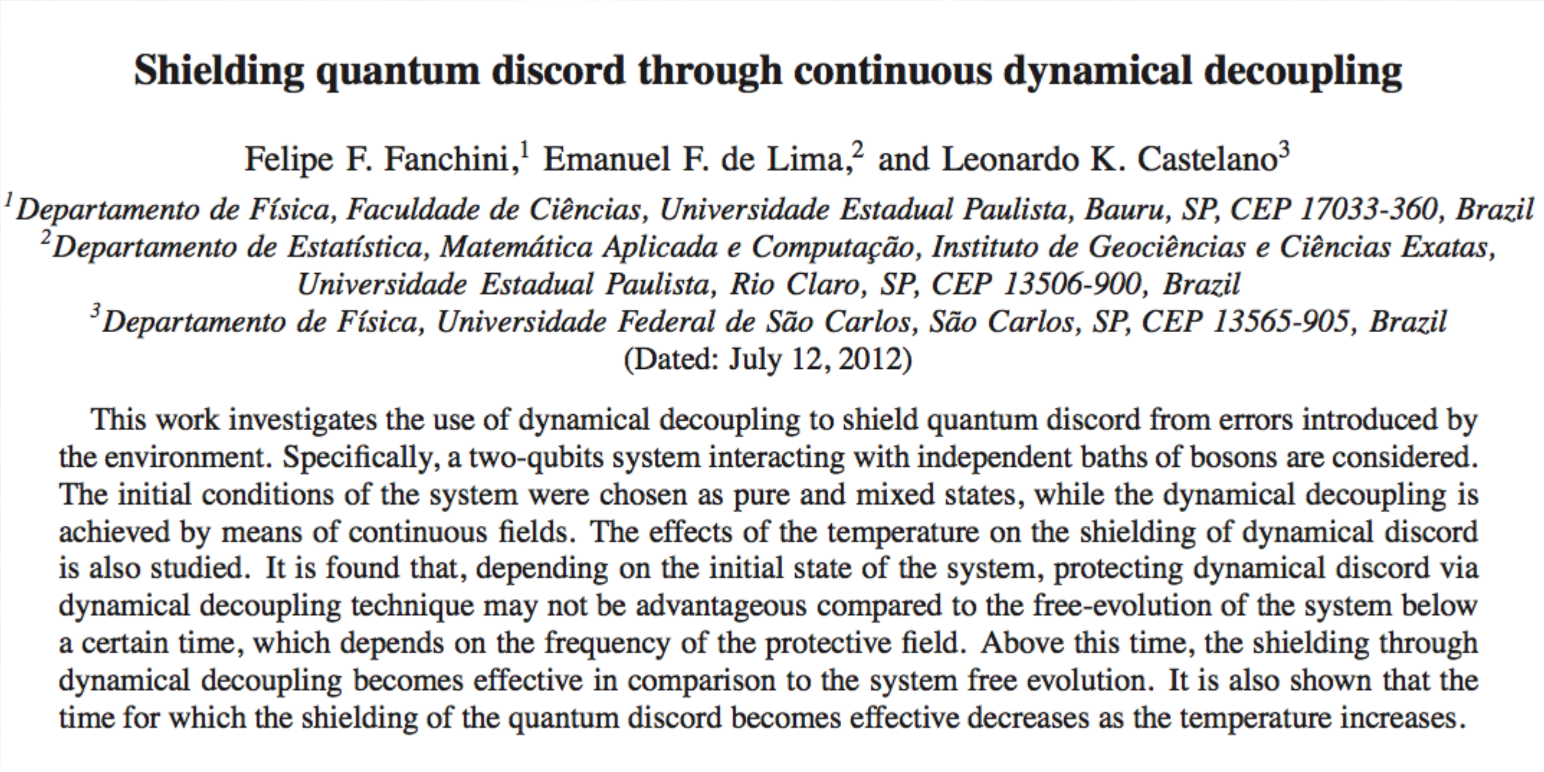


FIG. 1. (Colors online) Dynamics of mutual information (green dotted line), classical correlations (red dashed line) and quantum discord (blue solid line) as a function of γt for $c_1(0) = 1$, $c_2(0) = -c_3$ and $c_3 = 0.6$. In the inset we plot the eigenvalues λ_{Ψ}^{+} (blue solid line), λ_{Ψ}^{-} (green dash-dotted line), λ_{Φ}^{+} (red dashed line) and λ_{Φ}^{-} (violet dotted line) as a function of γt for the same parameters.



Shielding quantum discord through continuous dynamical decoupling

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This work investigates the use of dynamical decoupling to shield quantum discord from errors introduced by the environment. Specifically, a two-qubits system interacting with independent baths of bosons are considered. The initial conditions of the system were chosen as pure and mixed states, while the dynamical decoupling is achieved by means of continuous fields. The effects of the temperature on the shielding of dynamical discord is also studied. It is found that, depending on the initial state of the system, protecting dynamical discord via dynamical decoupling technique may not be advantageous compared to the free-evolution of the system below a certain time, which depends on the frequency of the protective field. Above this time, the shielding through dynamical decoupling becomes effective in comparison to the system free evolution. It is also shown that the time for which the shielding of the quantum discord becomes effective decreases as the temperature increases.

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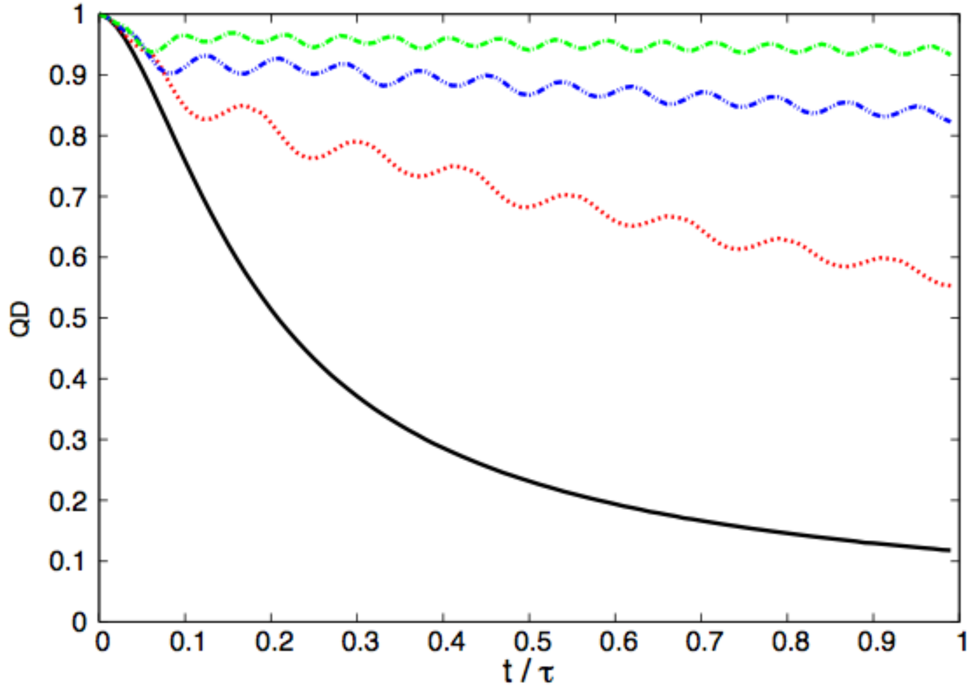


FIG. 2: (Color Online) Quantum Discord as a function of time considering a pure initial state $\rho^0_{AB}(0)$ and $T = 0$K. The solid (black) line gives the quantum discord when the control field is turned off and the dotted (red), double dot-dashed (blue), and dot-dashed (green) lines give the quantum discord when the control fields are turned on at $t = 0$, using $n_x = 2, 3, 4$ respectively.

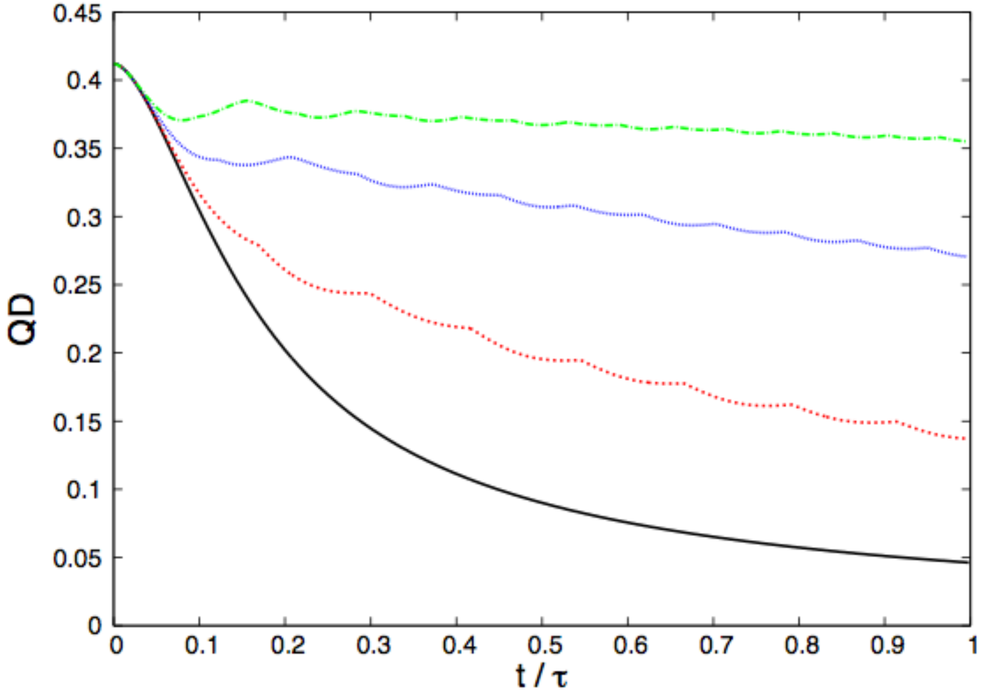


FIG. 3: (Color Online) Quantum Discord as a function of time considering a mixed initial state $\rho^1_{AB}(0)$ and $T = 0$K. The solid (black) line gives the quantum discord when the control field is turned off and the dotted (red), double dot-dashed (blue), and dot-dashed (green) lines give the quantum discord when the control fields are turned on at $t = 0$, using $n_x = 2, 3, 4$ respectively.

What could we say about the protection in this situation ???

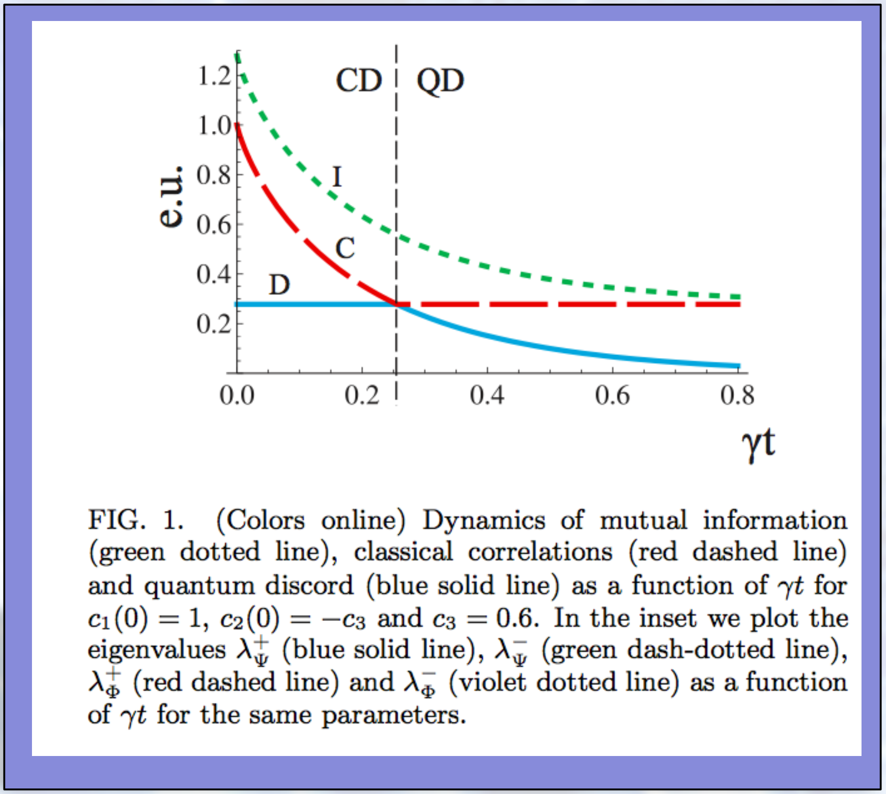
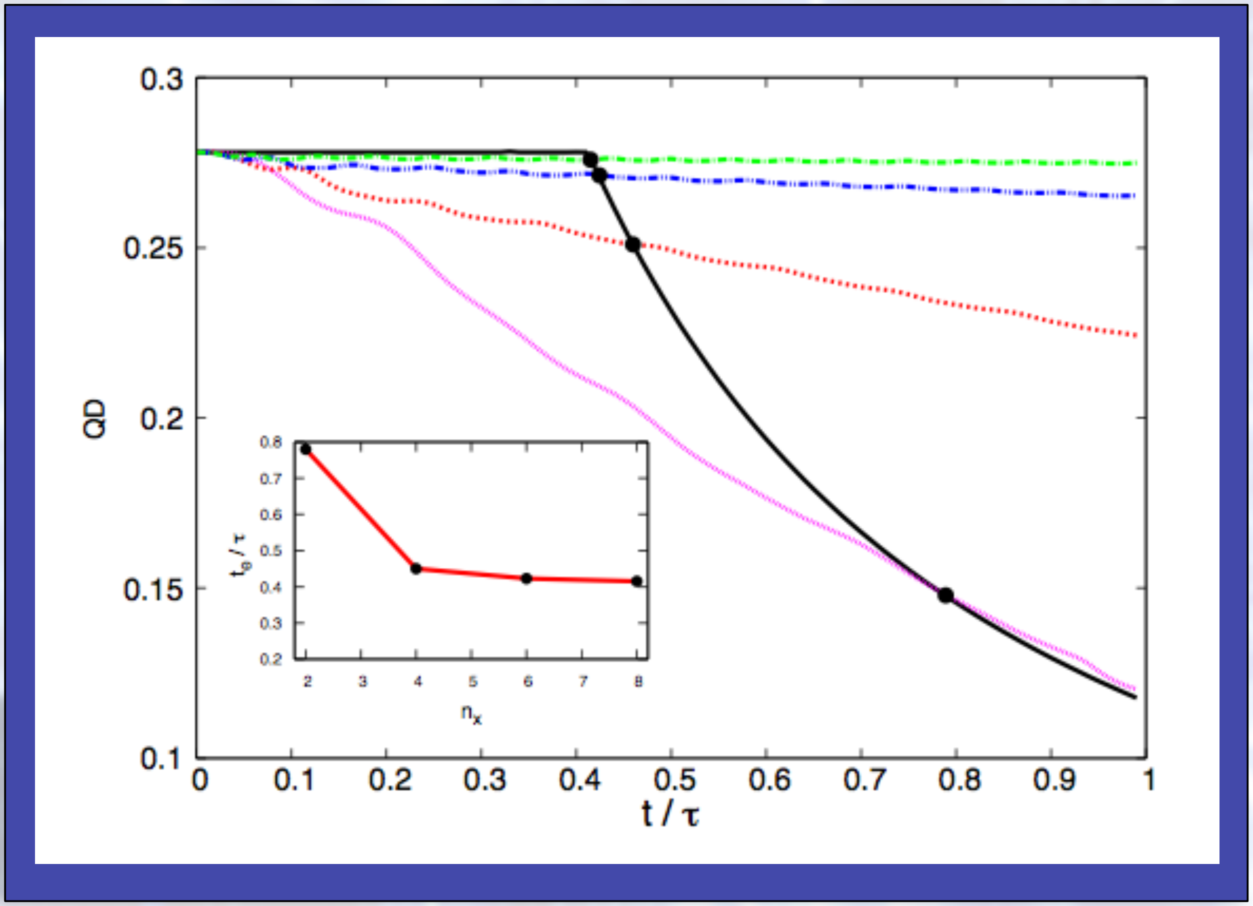
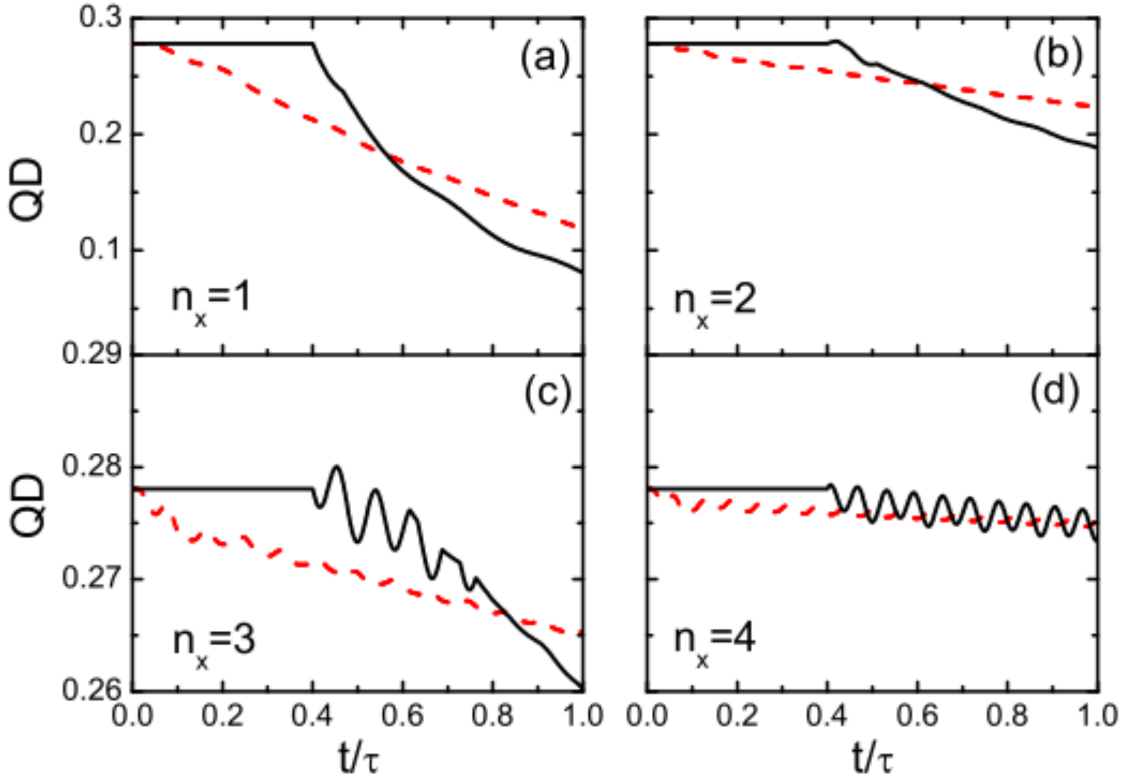


FIG. 1. (Colors online) Dynamics of mutual information (green dotted line), classical correlations (red dashed line) and quantum discord (blue solid line) as a function of γt for $c_1(0) = 1$, $c_2(0) = -c_3$ and $c_3 = 0.6$. In the inset we plot the eigenvalues λ_Ψ^+ (blue solid line), λ_Ψ^- (green dash-dotted line), λ_Φ^+ (red dashed line) and λ_Φ^- (violet dotted line) as a function of γt for the same parameters.


QD
0.3
0.25
0.2
0.15
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t / τ
t_θ / τ
0.8
0.7
0.6
0.5
0.4
0.3
0.2
2
3
4
5
6
7
8
n_x


(a)
(b)
(c)
(d)
QD
QD
0.3
0.2
0.1
0.29
0.28
0.27
0.26
n$_x$=1
n$_x$=2
n$_x$=3
n$_x$=4
0.0
0.2
0.4
0.6
0.8
1.0
t/τ
t/τ

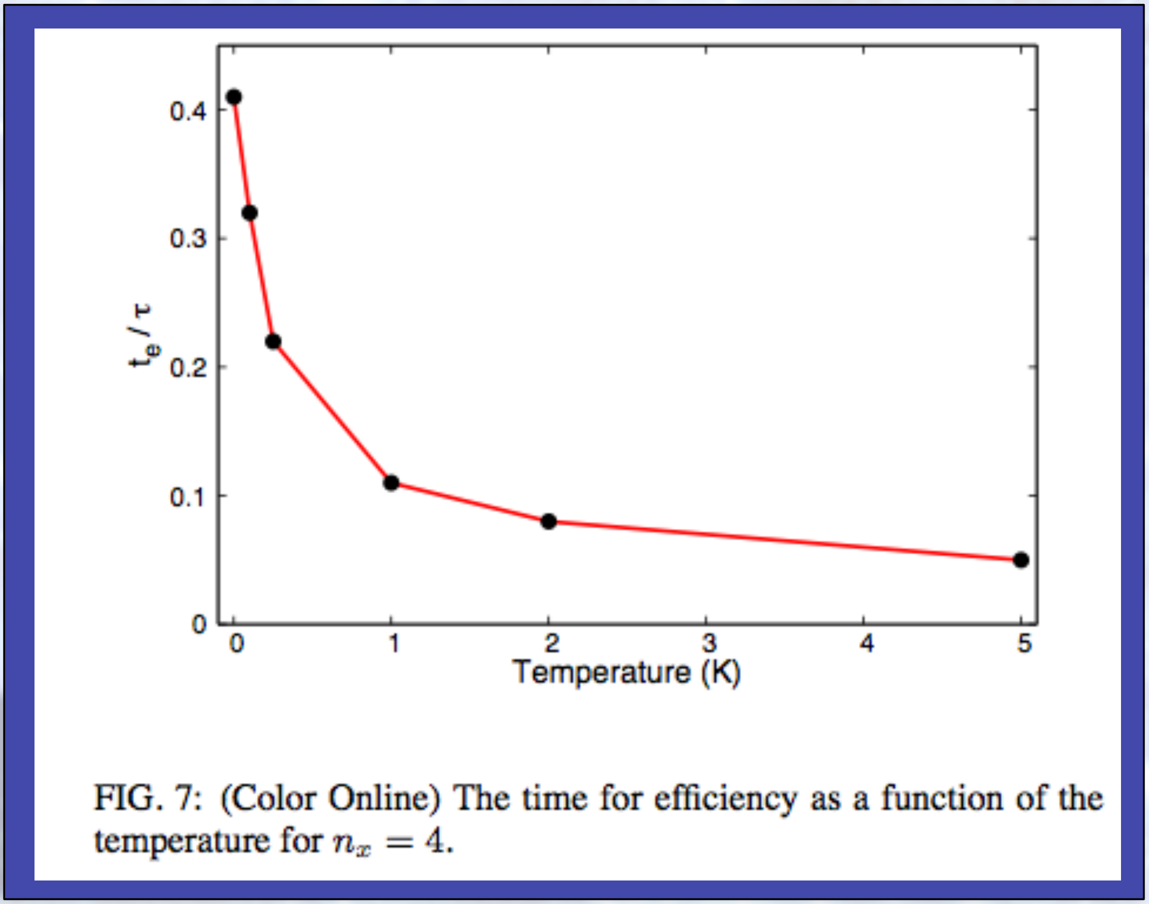


FIG. 7: (Color Online) The time for efficiency as a function of the temperature for $n_x = 4$.


"That's all Folks!"
THANK YOU VERY MUCH!!!