Cavity QED and Trapped Ions

E. Solano

UPV/EHU and Ikerbasque, Bilbao

Theory and exercises

Cali, September 2012

Chapter I: Atoms and EM fields

I.a) Brief review of quantum mechanics

I.b) Two-level atom driven by a classical field

I.c) Two-level atom driven by a quantized field

I.d) Microwave CQED versus optical CQED

I.e) Strongly-driven Jaynes-Cummings model

I.f) Second-order effective Hamiltonians

I.g) Selective interactions

I.a) Brief review of quantum mechanics

- The wavevector $|\Psi(t)\rangle$ describes the quantum state of a physical system at a given time t.

- The **Schrödinger equation**

$$i\hbar\frac{d}{dt}|\Psi(t)\rangle = H|\Psi(t)\rangle$$

is the dynamical equation that determines the temporal evolution of the wavevector $|\Psi(t)\rangle$ due to the Hamiltonian operator H, which describes the total energy of the system.

- If $H \neq H(t)$, then the formal solution of the Schrödinger equation is

$$|\Psi(t)\rangle = U(t_o,t)|\Psi(0)\rangle = e^{-iH(t-t_o)/\hbar}|\Psi(t_o)\rangle,$$

where $U(t_o,t) = e^{-iH(t-t_o)/\hbar}$ is the evolution operator associated with the time-independent Hamiltonian H.

- If $H = H(t)$, then the formal solution for the evolution operator is given trhough the **Dyson series**

$$\begin{aligned} U(t_o,t) = \ & 1 - \frac{i}{\hbar}\int_{t_o}^{t} H(t_o,t')dt' \\ & + \Big(-\frac{i}{\hbar}\Big)^2 \int_{t_o}^{t} dt' H(t_o,t') \int_{t_o}^{t'} H(t_o,t'')dt'' + \cdots, \end{aligned}$$

such that $|\Psi(t)\rangle = U(t_o,t)|\Psi(0)\rangle$. Recall that $U(t_o,t)$ is a unitary operator such that $U^\dagger U = UU^\dagger = 1$.

- The **density operator** $\rho(t) = |\Psi(t)\rangle\langle\Psi(t)|$ is an alternative description of the quantum state of a system and its dynamics is given by the von Neumann equation

$$\dot{\rho} = \frac{1}{i\hbar}[H, \rho].$$

- All physical predictions are the same if we use the Schrödinger equation or the von Neumann equation for $|\Psi\rangle$ or $\rho = |\Psi\rangle\langle\Psi|$, respectively.

- However, the Schrödinger equation allows only the description of **pure states**: the ones whose full information can be encoded in a ket $|\Psi\rangle$.

- When the available information about the system is reduced, we have to deal with **mixed states**, which can be described by density operators

$$\rho = \sum p_j |\Psi_j\rangle\langle\Psi_j|,$$

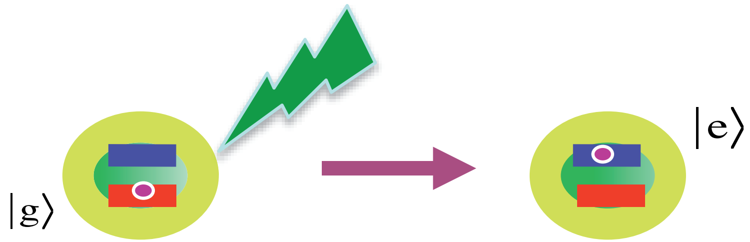
where p_j is the probability of finding $|\Psi_j\rangle$, $\sum_j p_j = 1$.

- The von Neumann equation is more general than the Schrödinger equation, allowing the description of noisy environments via **master equations**

$$\dot{\rho} = \frac{1}{i\hbar}[H, \rho] + \mathcal{L}\rho\,.$$

Here, the first term at the r.h.s. describes **the unitary or Hamiltonian evolution**, and the second term describes **the nonunitary or dissipative dynamics**.

I.b) Two-level atom driven by a classical field



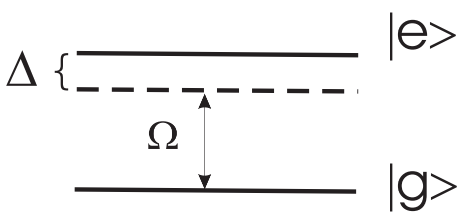
We consider a dipolar coupling between a two-level atom, with internal states $|g\rangle$ and $|e\rangle$, and a monochromatic coherent laser field $\vec{E} = \vec{E}_o \cos(\omega_L t + \phi)$,

$$\begin{aligned} H = \quad & H_o + H_{\rm dip} = \frac{\hbar\omega_o}{2}\sigma_z - \vec{d}.\vec{E} \\ = \quad & \frac{\hbar\omega_o}{2}\sigma_z + \hbar\Omega(\sigma^+ + \sigma^-)[e^{-i(\omega_L t+\phi)} + e^{i(\omega_L t+\phi)}], \end{aligned}$$

where $\sigma_z \equiv |e\rangle\langle e| - |g\rangle\langle g|$, $\vec{d} = e\vec{r}$ is the dipole operator, $\sigma^+ \equiv |e\rangle\langle g|$,$\sigma^- \equiv |g\rangle\langle e|$, and the dipolar coupling strength $\Omega \propto |\vec{E}_o|\langle g|r|e\rangle$. After the optical **rotating-wave-approximation (RWA)**, the Hamiltonian reads

$$H = \frac{\hbar\omega_o}{2}\sigma_z + \hbar\Omega[\sigma^+ e^{-i(\omega_L t+\phi)} + \sigma^- e^{i(\omega_L t+\phi)}]$$

with $\omega_o \equiv \omega_e - \omega_g$, and $\Delta \equiv \omega_o - \omega_L$.



In the interaction picture, with $U_o = e^{iH_o t/\hbar}$,

$$H^I = U_o H_{\mathsf{dip}} U_o^\dagger = \hbar(\Omega e^{-i\phi}\sigma^+ e^{i\Delta t} + \Omega e^{i\phi}\sigma^- e^{-i\Delta t})$$

In particular, under resonant conditions, $\Delta = 0$, and choosing $\phi = 0$,

$$H_x^I = \hbar\Omega\,\sigma_x,$$

where the Pauli operator $\sigma_x \equiv \sigma^+ + \sigma^-$. It is known that $|\pm\rangle \equiv (|g\rangle \pm |e\rangle)/\sqrt{2}$ are the eigenstates of σ_x with eigenvalues ± 1, such that $\sigma_x|\pm\rangle = \pm|\pm\rangle$. This **rotated basis** is sometimes called the (classical) dressed basis.

Always in resonance but with $\phi = \pi/2$,

$$H_y^I = \hbar\Omega\,\sigma_y,$$

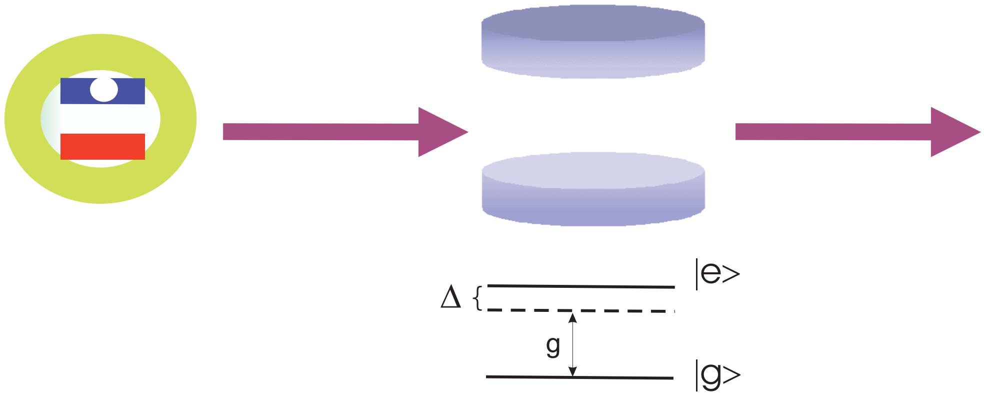
with $\sigma_y \equiv (\sigma^+ - \sigma^-)/i$.

In this way, by controlling the laser frequency ω_L and phase ϕ, we can realize arbitrary rotations around x-axis and y-axis, also called (classical) **Rabi oscillations**.

Exercise I.1: Calculate the temporal evolution of a general initial state $\alpha|\mathsf{g}\rangle + \beta|\mathsf{e}\rangle$ under a dynamics described by H, in the resonant ($\Delta = 0$), nonresonant ($\Delta \neq 0$), and dispersive ($|\Delta| \gg \Omega$) case.

Exercise I.2: Implement efficiently a series of $\{\sigma_x, \sigma_y, \sigma_z\}$ pulses to implement a Hadamard gate and an arbitrary phase gate. See Nielsen & Chuang, "Quantum information and quantum computation", and have a look at http://arxiv.org/abs/0908.3673.

I.c) Two-level atom driven by a quantized field



The Hamiltonian describing the coupling of a two-level atom and a single mode of the quantized electromagnetic field is

$$H = H_o + H_{\text{int}} = \frac{\hbar\omega_o}{2}\sigma_z + \hbar\omega a^\dagger a + \hbar g(\sigma^+ + \sigma^-)(a + a^\dagger)$$

where dipolar, two-level, and single-mode approximations have been done. Moreover, the RWA can be applied if $g/\omega \ll 1$, giving rise to the analytically solvable **Jaynes-Cummings (JC) model**,

$$H_{\text{JC}} = \frac{\hbar\omega_o}{2}\sigma_z + \hbar\omega a^\dagger a + \hbar g(\sigma^+ a + \sigma^- a^\dagger).$$

The JC eigenstates read

$$|+,n\rangle = \sin\theta_n|e\rangle|n\rangle + \cos\theta_n|g\rangle|n+1\rangle$$
$$|-,n\rangle = \cos\theta_n|e\rangle|n\rangle - \sin\theta_n|g\rangle|n+1\rangle$$

where

$$\begin{aligned}
\sin\theta_n &= \frac{2g\sqrt{n+1}}{\sqrt{(\Delta_n-\Delta)^2+4g^2(n+1)}} \\
\cos\theta_n &= \frac{\Delta_n-\Delta}{\sqrt{(\Delta_n-\Delta)^2+4g^2(n+1)}} \\
\Delta_n &= \sqrt{\Delta^2+4g^2(n+1)},
\end{aligned}$$

and the eigenvalues are

$$E_{\pm n} = \hbar\omega_o(n+\frac{1}{2}) \pm \frac{\hbar\Delta_n}{2}.$$

In the resonant case, $\Delta = 0$, the eigenstates are

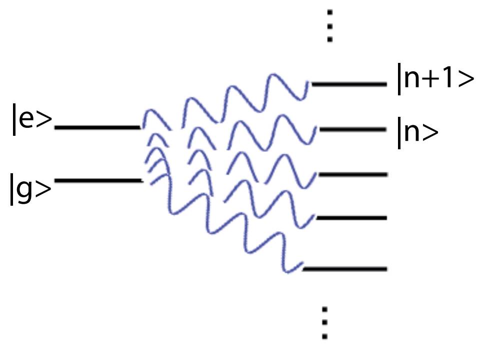
$$|+,n\rangle = \frac{1}{\sqrt{2}}(|e\rangle|n\rangle + |g\rangle|n+1\rangle)$$
$$|-,n\rangle = \frac{1}{\sqrt{2}}(|e\rangle|n\rangle - |g\rangle|n+1\rangle),$$

and are the dressed states of the JC model. The respective eigenvalues are $E_{\pm n} = \hbar\omega_o(n+\frac{1}{2}) \pm \hbar g\sqrt{n+1}$.

Exercise I.3: Show that under a suitable choice of phases, in resonance, and after an interaction time τ,

$$|e\rangle|n\rangle \to \cos(g\tau\sqrt{n+1})|e\rangle|n\rangle + \sin(g\tau\sqrt{n+1})|g\rangle|n+1\rangle$$
$$|g\rangle|n+1\rangle \to \cos(g\tau\sqrt{n+1})|g\rangle|n+1\rangle - \sin(g\tau\sqrt{n+1})|e\rangle|n\rangle$$

which are known as Rabi oscillations with Rabi frequency $g\sqrt{n+1}$.



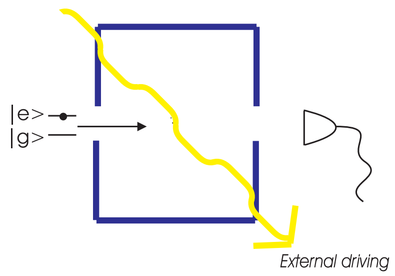
Exercise I.4: Show that for an atom initially in state $|\mathrm{e}\rangle$, and the field in an arbitrary pure state $\sum_n c_n|n\rangle$, the probability of finding the atom in the same state $|\mathrm{e}\rangle$ (survival probability) after an interaction time τ is

$$P_{\mathrm{e}}(\tau) = \frac{1}{2} + \frac{1}{2}\sum_n |c_n|^2 \cos(2g\tau\sqrt{n+1})$$

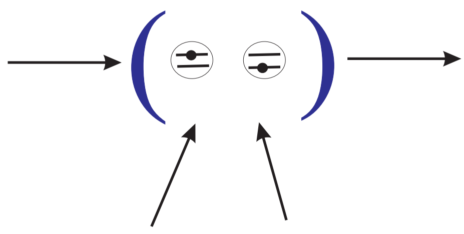
Is it possible to use this expression in order to extract the field populations $|c_n|^2$, in a set of experiments for different interactions times τ?

I.d) Microwave CQED versus optical CQED

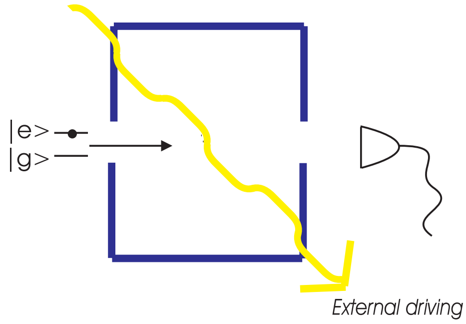
In the **microwave regime**, long-lived atomic levels and cavities are used. Typically, circular Rydberg levels live around 30 ms (Paris group) and a photon inside a high-quality microwave cavity, $Q \sim 10^{10}$, lives $\sim$ 0.1s (Garching and Paris groups). The coupling strength can reach $g = 50$ KHz and several coherent Rabi oscillations can happen.



In the **optical regime**, fast decaying optical transitions and high Q optical cavities are used. Typically, $Q \sim 10^6$, $g \leq 1$ MHz and three-level atoms are considered (reduced effectively to two levels) for inhibiting spontaneous emission. A great advantage is the possibility of having a controllable input and output axial field. When combined with trapped-ion technology, optical cavities represent a promising avenue for the implementation of quantum stations performing long-distance quantum communication.



I.e) Strongly-driven Jaynes-Cummings model



We consider a two-level atom coupled to a single-mode cavity and a classical driving acting transversally on the atom. The associated Hamiltonian reads

$$H = \frac{\hbar\omega_o}{2}\sigma_z + \hbar\omega a^\dagger a + \hbar g(\sigma^\dagger a + \sigma a^\dagger) + \hbar\Omega(\sigma^\dagger e^{-i\omega_L t} + \sigma e^{i\omega_L t}),$$

where Ω is the coupling between the external driving and the atoms and ω_L is the driving frequency.

We rewrite H in a frame rotating at the frequency of the driving field, applying the unitary transformation $U_o = e^{i(\frac{\omega_L}{2}\sigma_z + \omega_L a^\dagger a)t}$,

$$H^L = \frac{\hbar\Delta}{2}\sigma_z + \hbar\delta a^\dagger a + \hbar\Omega(\sigma^\dagger + \sigma) + \hbar g(\sigma^\dagger a + \sigma a^\dagger),$$

with $\Delta = \omega_o - \omega_L$ and $\delta = \omega - \omega_L$.

We go to the interaction picture with $\Delta = 0$, using

$$H_o^L = \hbar\delta a^\dagger a + \hbar\Omega(\sigma^\dagger + \sigma)$$
$$H_{\text{int}}^L = \hbar g(\sigma^\dagger a + \sigma a^\dagger),$$

obtaining

$$H^I = \frac{\hbar g}{2}(|+\rangle\langle+| - |-\rangle\langle-| + e^{2i\Omega t}|+\rangle\langle-| - e^{-2i\Omega t}|-\rangle\langle+|)ae^{-i\delta t} + \text{H. c.},$$

where $|\pm\rangle = \frac{1}{\sqrt{2}}(|g\rangle \pm |e\rangle)$ are the eigenstates of σ_x.

In the strong-driving regime, $\Omega \gg g$, and with $\delta = 0$,

$$\begin{aligned} H_{\text{eff}} &= \frac{\hbar g}{2}\big(|+\rangle\langle+| - |-\rangle\langle-|\big)(a + a^\dagger) \\ &= \frac{\hbar g}{2}(\sigma^\dagger + \sigma)(a + a^\dagger) = \frac{\hbar g}{2}\sigma_x(a + a^\dagger) \end{aligned}$$

and we obtain **simultaneous JC and anti-JC !!**

If the initial atom-field state is $|g\rangle|0\rangle = \frac{1}{\sqrt{2}}\left[|+\rangle + |-\rangle\right]|0\rangle$, then the evolution yields the state

$$\frac{1}{\sqrt{2}}\left[|+\rangle|\alpha\rangle + |-\rangle| - \alpha\rangle\right]$$

with $\alpha = -igt/2$. A measurement in the bare basis $\{|g\rangle, |e\rangle\}$ produces

$$\mathcal{N}^{\pm}\left[|\alpha\rangle \pm | - \alpha\rangle\right],$$

called **even/odd coherent states.**

At this point, it is good to review the theory of the displacement operator. I assume you know that

$$D(\alpha) = e^{\alpha a^\dagger - \alpha^* a},$$

such that a coherent state, the most classical of the quantum states, can be created from the vacuum $D(\alpha)|0\rangle = |\alpha\rangle$, *with*

$$|\alpha\rangle \equiv e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle.$$

I assume also that you can deduce (hidden exercise!) that Hamiltonian $H = \hbar(g^* a + g a^\dagger)$ *yields evolution operator* $U(t) = D(-igt)$.

Also in the strong driving limit $\Omega \gg g$ but with $\delta = \pm 2\Omega$

$$H_{JC}^{(+)} = \frac{\hbar g}{2}\big(|+\rangle\langle -|\, a + |-\rangle\langle +|\, a^\dagger\big)$$
$$H_{AJC}^{(-)} = \frac{\hbar g}{2}\big(|-\rangle\langle +|\, a + |+\rangle\langle -|\, a^\dagger\big),$$

JC or anti-JC in the dressed basis $|\pm\rangle$!!

For more details, have a look at PRL 90, 027903 (2003); Phys. Rev. A 71, 013811 (2005); PRA 77, 033839 (2008).

I.f) Second-order effective Hamiltonians

We assume that in a certain interaction picture it is possible to write a time-dependent Hamiltonian as

$$H^I(t) = \hbar \sum_j [A_j^\dagger e^{i\delta_j t} + A_j e^{-i\delta_j t}]$$

where $A_j^\dagger$ is a time-independent function of system operators. For example, $A_j^\dagger = g_j\, a_j^\dagger b_j^2 ... + ...$, where g_j are coupling strengths.

If $|\delta_j| \gg g_j$, $\forall j$, and $|\delta_j \pm \delta_k| \gg g_k$, $\forall \{j \neq k\}$, then, it can be shown that the Dyson series for the evolution operator associated with time-dependent Hamiltonian $H^I(t)$ can be recast in exponential form, $U = \exp(-iH_{\text{eff}} t/\hbar)$, where

$$H_{\text{eff}} = \hbar \sum_j \frac{[A_j^\dagger, A_j]}{\delta_j} \, .$$

There are several methods for deriving this kind of effective Hamiltonians. The present one, called the **commutator theorem**, can be derived using the Dyson series, and is very simple and powerful.

I will give two useful and simple examples.

Dispersive regime of the JC model

The Hamiltonian corresponding to the detuned JC model and in the interaction picture can be written as

$$H^I(t) = \hbar g\big(|\mathrm{e}\rangle\langle\mathrm{g}|a e^{i\delta t} + |\mathrm{g}\rangle\langle\mathrm{e}|a^\dagger e^{-i\delta t}\big),$$

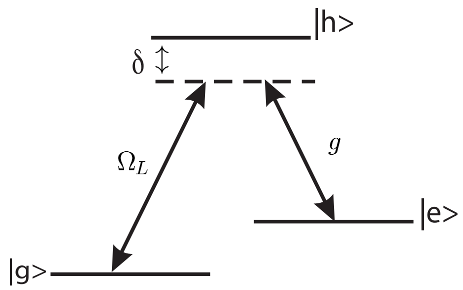
where δ is the frequency difference between the two-level system and the harmonic oscillator. Then, following the commutator theorem, with $|\delta| \gg g$ and $A^\dagger = g\sigma^\dagger a$, the second-order effective Hamiltonian is

$$H_{\text{eff}} = \hbar\frac{g^2}{\delta}[|\mathrm{e}\rangle\langle\mathrm{e}|(a^\dagger a + 1) - |\mathrm{g}\rangle\langle\mathrm{g}|a^\dagger a].$$

This shows that the dispersive regime produces, effectively, AC-Stark shift terms conditioned to the number of excitations in the oscillator.

Raman system

We will show how to realize the **adiabatic elimination** of the upper level of a three-level atom in Lambda configuration



The associated Hamiltonian in the interaction picture reads

$$H_I(t) = \hbar\Omega_L\big(|\mathsf{g}\rangle\langle\mathsf{h}|e^{i\delta t} + |\mathsf{h}\rangle\langle\mathsf{g}|e^{-i\delta t}\big) \\ +\hbar\mathsf{g}\big(|\mathsf{e}\rangle\langle\mathsf{h}|a^\dagger e^{i\delta t} + |\mathsf{h}\rangle\langle\mathsf{e}|a e^{i\delta t}\big).$$

Then, following the commutator theorem with $\delta \gg \{\Omega_L, g\}$ and $A^\dagger = \Omega_L|\mathsf{g}\rangle\langle\mathsf{h}| + g|\mathsf{e}\rangle\langle\mathsf{h}|a^\dagger$, the second-order effective Hamiltonian reads

$$H_{\mathsf{eff}} = \hbar\frac{\Omega_L^2}{\delta}|\mathsf{g}\rangle\langle\mathsf{g}| + \frac{g^2}{\delta}|\mathsf{e}\rangle\langle\mathsf{e}|a^\dagger a + \frac{\Omega_L g}{\delta}(|\mathsf{g}\rangle\langle\mathsf{e}|a + |\mathsf{e}\rangle\langle\mathsf{g}|a^\dagger),$$

where $\Omega_{\mathsf{eff}} = \Omega_L g/\delta$ is the effective Raman coupling associated with an anti-JC coupling, and the first and second terms of the r.h.s. are AC-Stark shifts.

In principle, the anti-JC effective coupling is not on resonance, but this fact can be corrected or exploited depending on what are our purposes. If we want to correct them, then, we should note that this cannot be done for all number states $|n\rangle$, due to the $a^\dagger a$ dependence. It can only be done for a given anti-JC doublet, say $\{|g\rangle|0\rangle, |e\rangle|1\rangle\}$.

Exercise I.5:

a) Derive in detail Hamiltonian

$$H = \frac{\hbar g}{2}\sigma_x(a + a^\dagger).$$

b) Find the conditions under which it is possible to implement a dispersive regime with

$$H = \frac{\hbar g}{2}\sigma_x(ae^{i\delta t} + a^\dagger e^{-i\delta t}),$$

another one with

$$H = \frac{\hbar g}{2}(\sigma^+ e^{-i\bar{\delta} t} + \sigma^- e^{i\bar{\delta} t})(a + a^\dagger),$$

and derive the associated effective Hamiltonians.

c) Comment your findings and results.

I.g) Selective interactions

The **Jaynes-Cummings** Hamiltonian in the interaction picture, where the atom-field system is resonant, reads

$$H^I_{\mathsf{JC}} = \hbar g(\sigma^\dagger a + \sigma a^\dagger)$$

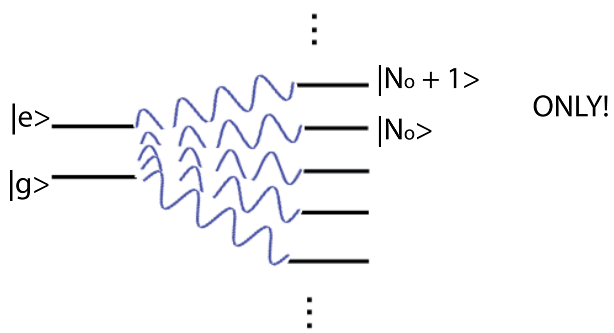
and yields oscillations in **all subspaces** of the kind

$$\{|g,n\rangle, |e,n-1\rangle\},\ \forall n = 0,1,...$$

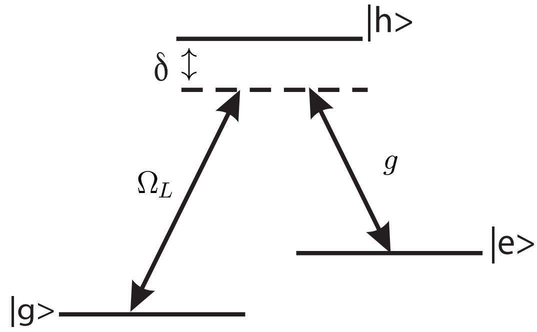
- We call here **selective interaction** to a resonant interaction inside a chosen subspace of the atom-field Hilbert space,

$$\{|g,N_o\rangle, |e,N_o \pm 1\rangle\}, \text{with fixed}\, N_o,$$

while all others remain dispersive.



A selective interaction in microwave CQED



We excite a three-level atom, in lambda configuration, with a classical field of frequency ω_L and a quantized mode of frequency ω_o, $\delta = \omega_{\mathsf{h}} - \omega_{\mathsf{g}} - \omega_L = \omega_{\mathsf{h}} - \omega_{\mathsf{e}} - \omega_o$.

It can be shown that, in the interaction picture, the Hamiltonian in the RWA can be written as

$$H_{\mathsf{int}} = \hbar\Omega_L\sigma_{\mathsf{hg}}e^{-i\delta t} + \hbar g\sigma_{\mathsf{he}}ae^{-i\delta t} + \mathsf{H.c.},$$

where $\sigma_{jm} \equiv |j\rangle\langle m|$ is an atomic flip operator and a the annihilation operator of the cavity mode.

If $|\delta| \gg \{|\Omega_L|, |g|\}$, the upper level $|\mathsf{h}\rangle$ can be adiabatically eliminated yielding an effective 2^{nd} order anti-JC Hamiltonian

$$\begin{aligned} H_{\mathsf{eff}} = \quad & \hbar\frac{\Omega_L^2}{\delta}\sigma_{gg} + \hbar\,\frac{g^2}{\delta}a^\dagger a\;\sigma_{ee} \\ & +\hbar\frac{g\Omega_L}{\delta}(\sigma^+a^\dagger + \sigma^- a)\,, \end{aligned}$$

with $\sigma^+ = \sigma_{\mathsf{eg}}$.

Associated with the transition in subspace

$$|g, N_o\rangle \longleftrightarrow |e, N_o + 1\rangle,$$

there is an effective detuning

$$\Delta_{eg}^{N_o} = \frac{g^2}{\delta}(N_o + 1) - \frac{\Omega_L^2}{\delta}$$

that can be cancelled by dc Stark shift for a given N_o.

Selectivity appears when we are able to tune to resonance a specific subspace transition

$$|g, N_o\rangle \overset{resonant}{\longleftrightarrow} |e, N_o + 1\rangle,$$

while all other doublet transitions remain dispersive.

When frequency adjustment is made for one specific subspace $\{|g, N_o\rangle, |e, N_o + 1\rangle\}$, remaining detunings associated with other subspaces ($n \neq N_o$) change to

$$\Delta_{eg}^{n\,*} \equiv \Delta_{eg}^{n} - \Delta_{eg}^{N_o} = \frac{|g|^2}{\delta}(n - N_o).$$

If after this reshifting process, $\forall n \neq N_o$,

$$\Delta_{eg}^{n\,*} \gg \frac{g\,\Omega_L}{\delta},$$

then we arrive to the **selectivity condition**

$$r \equiv \frac{g}{\Omega_L} \gg 1.$$

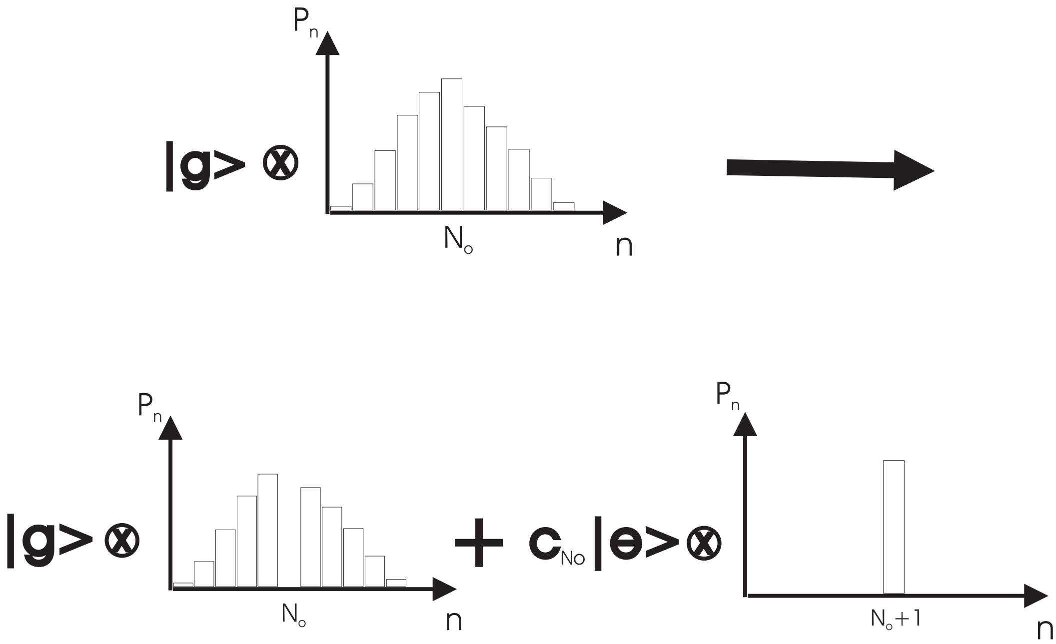
Large Fock state generation

We tune our system to be selectively resonant with the atom-field subspace

$$\{|g, N_o\rangle, |e, N_o + 1\rangle\}.$$

Then, we let it evolve for a time equivalent to a $\frac{\pi}{2}$-pulse, so that selectively the initial population

$$|g, N_o\rangle \longrightarrow |e, N_o + 1\rangle,$$



- When we measure the atom in $|e>$, the field is projected in Fock state $|N_o + 1\rangle$ with probability $P_e = |c_{N_o}|^2$.

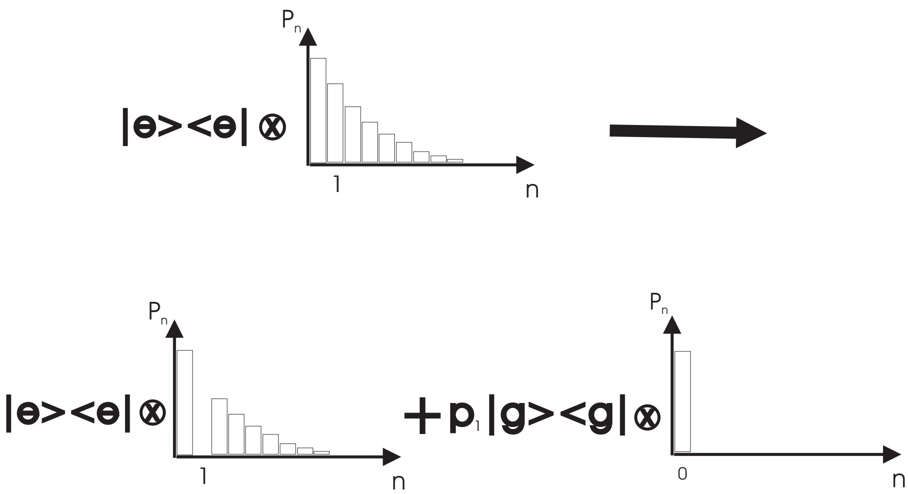
Selective cooling to the ground state

If now we tune to resonance the subspace

$$\{|g,0\rangle, |e,1\rangle\},$$

and the initial atom-field state is

$$|e\rangle \otimes \sum_n c_n |n>,$$



When we measure the atom in $|g\rangle$, with probability $|c_1|^2$, the field is projected onto the ground state $|0>$!!

A similar result is obtained from an initial thermal state or other statistical mixture.

Photon statistics and Wigner function

When subspace $\{|g, N_o\rangle, |e, N_o + 1\rangle\}$ was tuned to resonance, under the selectivity condition $r \gg 1$, we showed that after a $\frac{\pi}{2}$-pulse

$$P_e = |c_{N_o}|^2 \equiv P_{N_o},$$

which means that the measurement of P_e is equivalent to the measurement of the probability of finding N_o photons in the initial field, P_{N_o}.

In this way, we can measure the complete photon statistics P_n of an initial unknown field ρ, pure or mixed.

Combined with the possibility of displacing the unknown field, this method allows the fully reconstruction of its Wigner function through

$$W(-\alpha) = 2 \sum_n (-1)^n P_n(\alpha),$$

where $P_n(\alpha) = \langle n| D(\alpha)\rho D^{-1}(\alpha) |n\rangle$ is the number distribution of state ρ displaced coherently in the phase space by α.

For more details, have a look at PRL 87, 093601 (2001).

Exercise I.6:

a) Read carefully the last section (I.g) about "Selective interactions in microwave CQED".

b) Rederive on your onw the selective effective Hamiltonian and the selective condition.

c) Can you figure out other potential applications of selective interactions?

d) Could we implement selective interactions in circuit QED?