# Quantum ontological models and the PBR theorem

A. F. Ducuara
C. E. Susa
J. H. Reina

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Outline

- Quantum ontological models (QOMs)
  - $\psi$-onticity.
  - Factorisability.
  - Not Post-Peierls compatibility ($\neg PP$ compatibility).
- PBR theorem.
- An example.
- The same example under noisy channel.

# Schematic representation

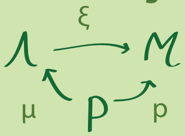
PM ⟶ P, M

QM Interpretations
Hidden Variables
EPR “Paradox”
Bell Theorem

Measure Problem
V Postulate

## Ontological Models (OMs)

Ontic States Space




Measure Space

Epistemic States Space

$\lambda \in \Lambda$

$p \in P$

$\xi \in M$

- Conditional Probabilities. $\mu(\lambda|p), \xi(k|\lambda), p(k|p)$
- Total Probability Law. $\int_\Lambda \mu_p(\lambda)\xi_\lambda(k)d\lambda = p_p(k)$

## Classical Ontological Model (COM)

Λ Classical Mechanics

PM Classical Statistical Mechanics

- Conditional Probabilities.

## Quantum Ontological Models (QOMs)

+ 3 assignments

# Quantum Ontological Models (QOMs) [1]

Quantum physical system represented by a Hilbert space $H$.

## Quantum mechanics on OMs:

- $P$: set of density matrices $p \to p_\psi$
- $M = \{E_k\}$ Positive-Operator Valued Measure (POVM). $k \to E_k$
- $p(E_k|\rho_\psi) := tr(E_k \rho)$, Born rule on QOMs (P, M)

$$\Lambda = ?$$

$$\xi(E_k|\lambda) = ?, \quad \mu(\lambda|\rho_\psi) = ?$$

Quantum mechanics interpretations

Total probability law: $\forall E_k \in M, \rho_\psi \in P$

$$\int_\Lambda \mu_{\rho_\psi}(\lambda)\xi_\lambda(E_k)d\lambda = p_{\rho_\psi}(E_k) = tr(E_k \rho)$$

[1] N. Harrigan, R. W. Spekkens. Einstein, incompleteness, and the epistemic view of quantum states arXiv:0706.2661. 2007.

# Why are QOMs important ?

On QOMs we have:

- EPR Completeness 1935 [2].
- BELL Locality 1964.
- Contextuality [3].

The arguments about these properties can be reconstructed.

QOMs were introduced five years ago.

$\psi$ onticity.

[2] N. Harrigan, R. W. Spekkens. Einstein, incompleteness, and the epistemic view of quantum states arXiv:0706.2661. 2007.

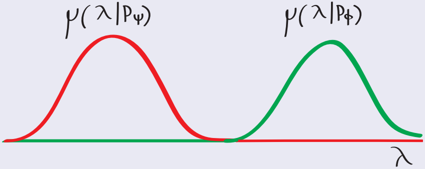
[3] N. Harrigan, T. Rudolph. Ontological models and the interpretation of contextuality arXiv:0709.4266v1. 2007.

Quantum physical system $H$, that can be modeled by a QOM $(\Lambda, M, P, \{\mu(\lambda|\rho_\psi)\}, \{\xi(E_k|\lambda)\}, \{p(E_k|\rho_\psi)\})$

## $\psi$-ontic QOM $(P, \Lambda)$

$\forall \lambda \in \Lambda, \forall \rho_\psi, \rho_\phi \in P, \rho_\psi \neq \rho_\phi$

$$\boxed{\mu(\lambda|\rho_\psi)\mu(\lambda|\rho_\phi) = 0}$$




All probabilities have pairwise disjoint support.

## Realism

$\psi$-complete QOMs $\subset$ $\psi$-ontic QOMs

# Two properties

## Factorisability $(P, \Lambda)$

In a multipartite system $\vec{\lambda} = (\lambda_1, ..., \lambda_n) \in \Lambda^n$, $\rho_\Psi \in P$

$$\mu_\Psi(\vec{\lambda}) = \mu_{\psi_1}(\lambda_1)...\mu_{\psi_n}(\lambda_n) \qquad \mu_{\psi_i}(\lambda_i) = \mu(\lambda_i|\rho_{\psi_i})$$

- Factorisability can be argued through separability of $\rho_\Psi \in P$.

## Not PP compatibility [4] $(\neg PP)$ (P, M)

$$\forall E_k \in M, \exists \rho_k \in P \text{ such that } p(E_k|\rho_\psi) = tr(E_k \rho_k) = 0$$

- Showing $\neg PP$ is not trivial.
- $\neg PP$ is an unnatural property.

[4] C. A. Fuchs C. M. Caves and R. Schack. Conditions for compatibility of quantum state assignments. PRA, 66, 2002.

# The PBR theorem [5] [6] [7]

Quantum physical system: $H = \bigotimes_{i=1}^{n} H_i \quad dim(H_i) = k_i$. Modeled by a QOM $(\Lambda, P, M)$. If the QOM satisfies:

- $\neg PP$ compatibility.
- $P = \{\rho_{\vec{x}}\}$ formed by separable states. $\rightarrow \mu_{\vec{x}}$ satisfies factorisability.

$\longrightarrow$ $\psi$-ontic QOM.

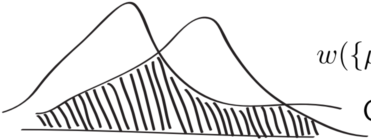
A result from the last year.

[5] M. F. Pusey, J. Barrett, and T. Rudolph. The quantum state cannot be interpreted statistically arXiv:1111.3328. 2011.

[6] M. J. W. Hall. Generalisations of the recent Pusey-Barrett-Rudolph theorem for statistical models of quantum phenomena arXiv:1111.6304. 2011.

[7] M. F. Pusey, J. Barrett, and T. Rudolph. On the reality of the quantum state. Nature Phys., 8:476, 2012.

# Schematic proof



$$w(\{\mu_{\vec{x}}\}) := \int_{\Lambda^n} min_{\vec{x}}\{\mu_{\vec{x}}(\vec{\lambda})\}d\vec{\lambda}$$

Overlap between probabilities

$$\boxed{w(\{\mu_{\vec{x}}\}) = 0 \text{ iff } \psi\text{-Ontic QOM}}$$

From the total probability law and $\neg PP$. For each $E_{\vec{x}}$ POVM element we have the appropriate $\rho_{\vec{x}}$ such that we have:

$$\int_{\Lambda^n} \mu_{\vec{x}}(\vec{\lambda})\xi_{\vec{\lambda}}(E_{\vec{x}})d\vec{\lambda} = tr(E_{\vec{x}}\rho_{\vec{x}}) = 0$$

$$\sigma(\{\rho_{\vec{x}}\}) := \sum_{\vec{x}} tr(E_{\vec{x}}\rho_{\vec{x}})$$

With factorisability, we can prove:

$$w(\{\mu_{\vec{x}}\}) \leq \sigma(\{\rho_{\vec{x}}\})$$

$$0 \leq w(\{\mu_{\vec{x}}\}) \leq \sigma(\{\rho_{\vec{x}}\}) = 0$$

$$w(\{\mu_{\vec{x}}\}) = 0 \quad \square$$

# Numerical approach

Given a space $P = \{\rho_{\vec{x}}\}$ formed by separable states.
If $\sigma(\{\rho_{\vec{x}}\}) = 0$ then the system satisfies $\neg PP$.
Then by PBR theorem the system can be represented by a $\psi$-ontic QOM.

Given a space $P = \{\rho_{\vec{x}}\}$ formed by separable states:

The problem is to minimize $\sigma(\{\rho_{\vec{x}}\})$ on $\{E_{\vec{x}}\}$ POVM.

$$\sigma(\{\rho_{\vec{x}}\}) := \sum_{\vec{x}} Tr\left(E_{\vec{x}}\rho_{\vec{x}}\right)$$

Matlab packages: SDPT3 [8], YALMIP [9].

[8] Michael J. Todd Kim-Chuan Toh and Reha H. Tutuncu. SDPT3 version 4.0 a MATLAB software for semidefinite-quadratic-linear programming

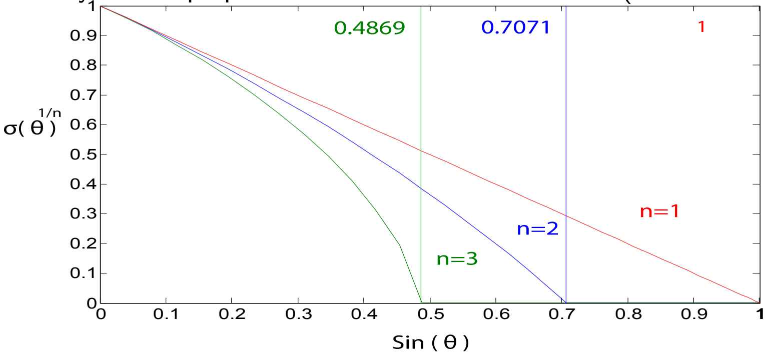
[9] J. Lofberg. YALMIP.

# Example I

$$|\Psi(\vec{x})\rangle = \bigotimes_{i=1}^{n} |\psi(x_i)\rangle\,, \qquad x_i = 0 \text{ or } 1, \qquad 0 \leq \theta \leq \pi/2$$

$$|\psi(x_i)\rangle = cos(\theta/2)\,|0\rangle + (-1)^{x_i} sin(\theta/2)\,|1\rangle$$

Each subsystem is prepared in the same kind of state. ( the same $\theta$)



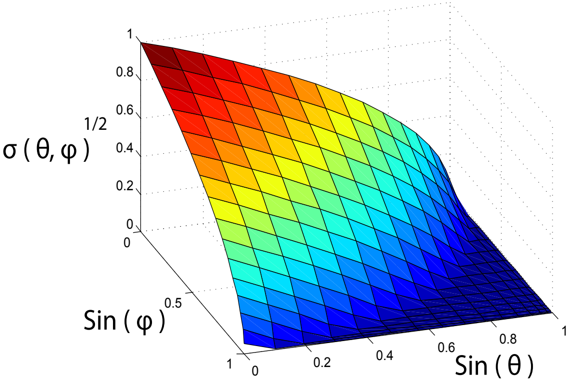

$$\theta \geq 2arct\left(2^{\frac{1}{n}} - 1\right)$$

Good analytical bound! but show ¬PP is not trivial.

# Example II, n=2 Qubits with general angles

$$|\psi(x_1)\rangle = cos(\theta/2)\,|0\rangle + (-1)^{x_1} sin(\theta/2)\,|1\rangle\,, \quad x_1 = 0 \text{ or } 1, \quad 0 \leq \theta \leq \pi/2$$
$$|\psi(x_2)\rangle = cos(\phi/2)\,|0\rangle + (-1)^{x_2} sin(\phi/2)\,|1\rangle\,, \quad x_2 = 0 \text{ or } 1, \quad 0 \leq \phi \leq \pi/2$$

Each subsystem is prepared in different kind of state. ( angles $\theta$, $\phi$)




We can play with the systems configurations.

# Example III: System under a noisy channel

The same example, but now the system can interacts with a quantum noise channel.

$$\rho'_{\vec{x}}(\theta) = \sum_k E_k \rho_{\vec{x}}(\theta) E_k^\dagger = p\rho + (1-p) E_1 \rho_{\vec{x}}(\theta) E_1^\dagger$$

$p$ is the probability of that the environment does not interacts with the system.

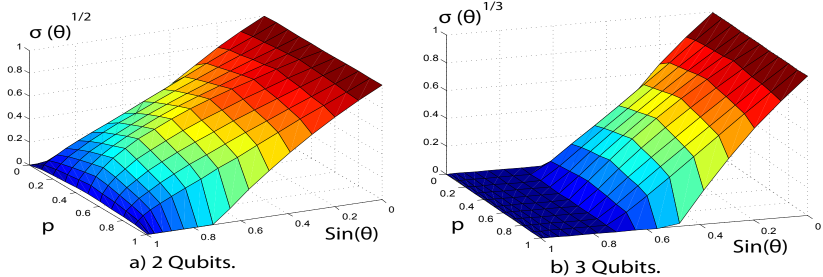



Figure: $E_0 = \sqrt{p}\mathbb{1}, E_1 = \sqrt{1-p}Y$, $0 \leq \theta \leq \pi/2$, $0 \leq p \leq 1$

We found similar results to X , Z.

We can explain in analitical form the reduction of the zero region.

The PBR theorem holds even for noisy scenarios ( 3 Qubits).

# Few conclusions

- OMs and QOMs are general theoretical frameworks to tackle realism on quantum mechanics; in particular, discussions like EPR "paradox", Bell theorem and contextuality can be reconstructed.

## The PBR theorem

- $\neg PP$ compatibility.
- $P$ formed by separable states.

$\longrightarrow$ $\psi$-ontic OM.

- PBR theorem states a relation between separability and $\psi$-onticity, properties that represent certain realism level.
- The PBR theorem gives us a tool to calculate a numerical approach to $\psi$-ontic realism through ¬PP (a non natural property) in separable states.
- In the particular example, if we consider an environment that interacts with the system, the PBR theorem holds for some scenarios.

# Working progress, perspectives

- Study better the $\neg$PP property.
- Work with separable no pure systems.
- There exist other proof with other property instead factorisability. Systems with $\psi$-ontic realism with entanglement?

Thanks.