Entanglement of identical particles.

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Abstract.

We have studied, how measuring device modifies the degree of entanglement in a system with two identical particles, and their statistics is controlled by the δ parameter, bosons $\delta = 1$ and fermions $\delta = -1$. The concurrence $C(|\Psi\rangle)$ characterizes the degree of entanglement in the system of two particles. When the states of one-particle are orthonormal, the concurrence is invariant under symmetrization or antisymmetrization.

Keywords: quantum information, identical particles, detection process, and entanglement

Many authors have studied the entanglement of identical particles

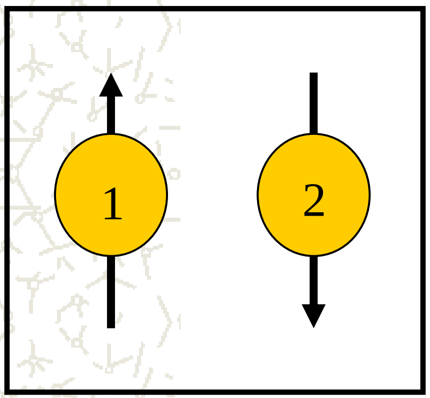
1. J. Schliemann et al., Phys. Rev. A **64**, 022303 (2001).
2. Li Y S et al., Phys. Rev. A **64**, 054302 (2001).
3. P. Zanardi, Phys. Rev. A **65**, 042101 (2002).
4. Shi Y, Phys. Rev. A **67**, 024301 (2003).
5. V Vedral, Cent. Eur. J. Phys. **1**, 197 (2003).
6. H M Wisemann et al., Phys. Rev. Lett. **91**, 010404 (2003).
7. Nikola Paunkovic, **Thesis of PhD:** The Role of Indistinguishability of Identical Particles in Quantun Information Processing, University of Oxford, **Advisor:** V. Vedral and A. Eckert (2004).
8. Cavalcanti D et al., Phys. Rev. A **72**, 062307 (2005).
9. L. Amico et al., Rev. of Mod. Phys. **80**, 517 (2008).
10. R. Horodecki, Rev. of Mod. Phys. **81**, 865 (2009).
11. Malte Tichy, **Thesis of PhD:** Entanglement and Interference of Identical Particles: University of Freiburg, **Advisor:** A. Buchleitner.
12. M. Tichy et al., J. Phys. B: At. Mol. Opt. Phys. **44** 192001 (2011).
13. O. Rendon et al. J. Phys.: Conf. Ser. **388** 012015 (2012).
14. M. Tichy et al., Fortshr. Phys., 1-13 (2012).
15. are not all…

Outlook for…

- What's entanglement between two distinguishable qubits (particles)?
  Only qualitative aspect and pure quantum states. (5.0 min)
- What's entanglement between two indistinguishable qubits (particles)?
  Only qualitative aspect and pure quantum states. (5.0 min)
- Do You can distinguish between quantum states?
  If It is true, What´s established condition? (7.0min)
- When do the identical particles can be distinguishable? (6.0 min)
- How measuring device distinguishes identical particles and to create entanglement? (6.0 min)

Qualitative properties of entangled pure state:

We have one system of two distinguishable **qubits** with spin:



$$|\Psi\rangle_D = \underbrace{|\uparrow\rangle}_{\text{qubit 1}} \otimes \overbrace{|\downarrow\rangle}^{\text{qubit 2}}$$

It isn't an entangled state.

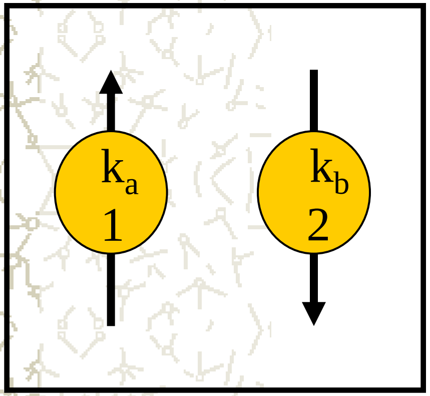
Graphic representation

$$|\Psi\rangle_D = \cos\frac{\theta}{2}|\uparrow\rangle \otimes |\downarrow\rangle + \sin\frac{\theta}{2}\; e^{i\phi}|\downarrow\rangle \otimes |\uparrow\rangle$$

More general state

Qualitative properties of entangled pure state:

We have one system of two distinguishable **particles** with spin:



Graphic representation

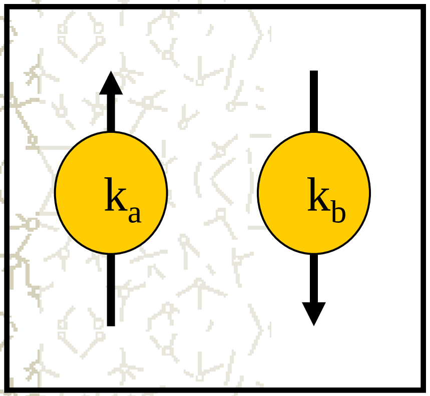
$$|\Psi\rangle_D = \underbrace{|k_a \uparrow\rangle}_{\text{Particle 1}} \otimes \overbrace{|k_b \downarrow\rangle}^{\text{Particle 2}}$$

It isn't an entangled state.

$$|\Psi\rangle_D = \cos\frac{\theta}{2}|k_a \uparrow\rangle \otimes |k_b \downarrow\rangle + \sin\frac{\theta}{2}\, e^{i\phi}|k_a \downarrow\rangle \otimes |k_b \uparrow\rangle$$

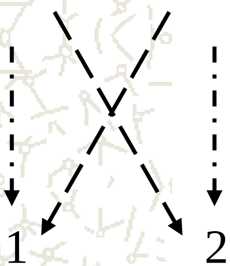
Now, We have two indistinguishable…

To see: J. Schielmann, D. Loss, and A. H. Mac Donald, Phys. Rev. B, **63**, 085311 (2001).



$$\left| k_a, \uparrow; k_b, \downarrow \right\rangle_{Slater} = \frac{1}{\sqrt{2}} \left\{ \left| k_a, \uparrow; k_b, \downarrow \right\rangle - \left| k_b, \downarrow; k_a, \uparrow \right\rangle \right\}$$

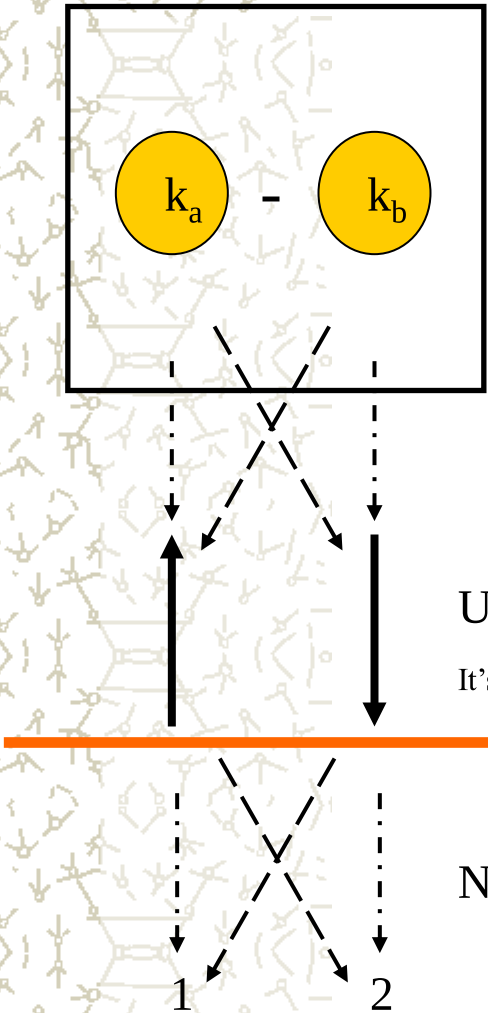
$$\left| k_a, \uparrow; k_b, \downarrow \right\rangle_{Slater} = \hat{a}^{+}_{k_a,\uparrow} \hat{a}^{+}_{k_b,\downarrow} \left| 0 \right\rangle$$



No useful entanglement

It isn’t an entangled state.

Let us look at a singlet state:



$$|s\rangle_I = \frac{1}{\sqrt{2}}\left\{|k_a,\uparrow;k_b,\downarrow\rangle_{Slater} - |k_a,\downarrow;k_b,\uparrow\rangle_{Slater}\right\}$$

$$|s\rangle_I = \frac{1}{\sqrt{2}}\left\{\hat{a}^+_{k_a,\uparrow}\hat{a}^+_{k_b,\downarrow} - \hat{a}^+_{k_a,\downarrow}\hat{a}^+_{k_b,\uparrow}\right\}|0\rangle$$

Useful entanglement

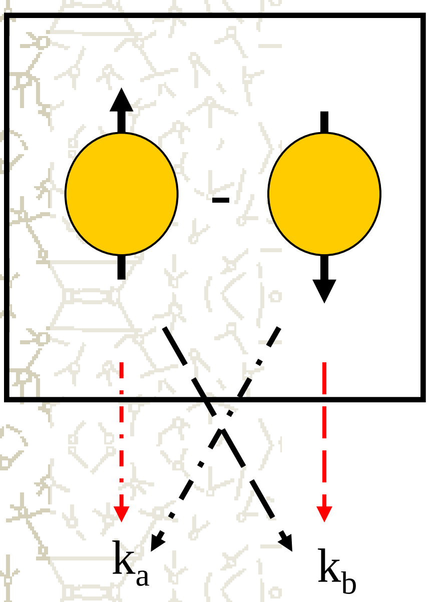
It's entangled with respect to spin degrees of freedom.

No useful entanglement

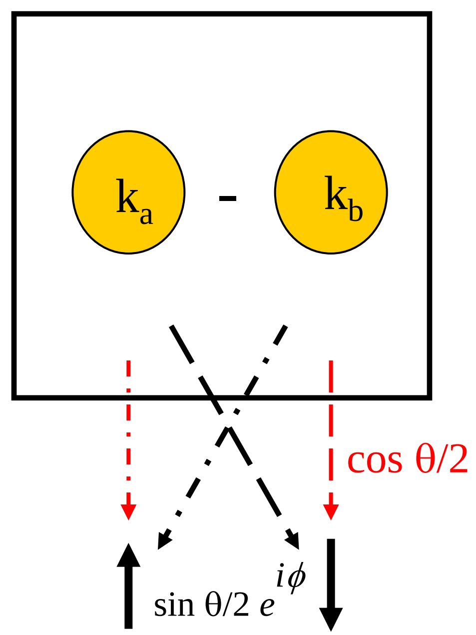
Other singlet state more general…

$$\left|s_G\right\rangle_I = \frac{1}{\sqrt{2}}\left\{\cos\frac{\theta}{2}\left|k_a,\uparrow;k_b,\downarrow\right\rangle_{Slater} + \sin\frac{\theta}{2}\, e^{i\phi}\left|k_a,\downarrow;k_b,\uparrow\right\rangle_{Slater}\right\}$$

$$\left|s_G\right\rangle_I = \frac{1}{\sqrt{2}}\left\{\cos\frac{\theta}{2}\quad \hat{a}^{+}_{k_a,\uparrow}\hat{a}^{+}_{k_b,\downarrow} + \sin\frac{\theta}{2}\, e^{i\phi}\quad \hat{a}^{+}_{k_a,\downarrow}\hat{a}^{+}_{k_b,\uparrow}\right\}\left|0\right\rangle$$



or



M. Tichy et al., Fortshr. Phys., 1-13 (2012).

Do You may distingish between quantum states?

- Well, there is theorem in information quantum about to the problem of distinguishing quantum states.
- Non-orthogonal quantum states are indistinguishables states, It´s the heart of quantum information and quantum informatiion.
- **Formal proof to see:** Nielsen and Chuang, Quantum Computation and Quantum Information.

- There is no process allowed by quantum mechanics that well reliably distingish between non-orthogonal states.

When do the identical particles can be distinguishable?

$$|s_G\rangle_I = \frac{1}{\sqrt{2}}\left\{\cos\frac{\theta}{2}|k_a,\uparrow;k_b,\downarrow\rangle_{Slater} + \sin\frac{\theta}{2}e^{i\phi}|k_a,\downarrow;k_b,\uparrow\rangle_{Slater}\right\}$$

- When in one degree of freedom of the system each particle have different eigenvalue.
- We invoke the above theorem about distinguishable orthogonal quantum state.
- Same results to see: Herbut Am. J. Phys. 2001

How measuring device distinguishes identical particles and to create entanglement.

$$|s_G\rangle_I = \frac{1}{\sqrt{2}}\left\{\cos\frac{\theta}{2}\,|k,\uparrow;k,\downarrow\rangle_{Slater} + \sin\frac{\theta}{2}\,e^{i\phi}|k,\downarrow;k,\uparrow\rangle_{Slater}\right\}$$

Lets take an operator of selective measure M_m, projector of the position self-ket.

$$M_m = \int_{B_1(x_A,r_A)} |x_1\rangle dx_1 \langle x_1| \otimes \int_{B_1(x_B,r_B)} |x_2\rangle dx_2 \langle x_2|,$$

with

$$B_1(x_A,r_A) = \left(x_A - {r_A}/{2},\, x_A + {r_A}/{2}\right),$$

and

$$B_1(x_B,r_B) = \left(x_B - {r_B}/{2},\, x_B + {r_B}/{2}\right).$$

Conclusion.

Well, the entanglement may be produced when you distinguish identical particles via measurement.