Exact dynamics of single qubit gate fidelities under the measurement-based quantum computation scheme

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Outline

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1. Introduction
2. One-way quantum computer
3. Exact dissipative dynamics
4. Oscillatory fidelity dynamics
5. Fidelity dynamics in an MBQC
6. Conclusion

Introduction

PHYSICAL REVIEW A **00**, 002300 (2012)

Exact dynamics of single-qubit-gate fidelities under the measurement-based quantum computation scheme

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(Received 22 January 2012; revised manuscript received 2 August 2012; published xxxxx)

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- this approach reveals that this characteristic depends only on the geometry of the state,
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that is:

under the action of a common dephasing environment, this nonmonotonical time dependence can provide us with appropriate time intervals for the preservation of better computational fidelities.

Outline

One-way quantum computation

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VOLUME 86, NUMBER 5 PHYSICAL REVIEW LETTERS 29 JANUARY 2001

Persistent Entanglement in Arrays of Interacting Particles

Hans J. Briegel and Robert Raussendorf

Sektion Physik, Ludwig-Maximilians-Universität, Theresienstrasse 37, D-80333 München, Germany

(Received 11 April 2000; revised manuscript received 28 August 2000)

We study the entanglement properties of a class of N-qubit quantum states that are generated in arrays of qubits with an Ising-type interaction. These states contain a large amount of entanglement as given by their Schmidt measure. They also have a high *persistency of entanglement* which means that $\sim N/2$ qubits have to be measured to disentangle the state. These states can be regarded as an entanglement resource since one can generate a family of other multiparticle entangled states such as the generalized Greenberger-Horne-Zeilinger states of $<N/2$ qubits by simple measurements and classical communication.

DOI: 10.1103/PhysRevLett.86.910 PACS numbers: 03.67.Lx, 03.65.Ta, 32.80.Pj

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DOI: 10.1103/PhysRevLett.86.910 PACS numbers: 03.67.Lx, 03.65.Ta, 32.80.Pj

VOLUME 86, NUMBER 22 PHYSICAL REVIEW LETTERS 28 MAY 2001

A One-Way Quantum Computer

Robert Raussendorf and Hans J. Briegel

Theoretische Physik, Ludwig-Maximilians-Universität München, Germany

(Received 25 October 2000)

We present a scheme of quantum computation that consists entirely of one-qubit measurements on a particular class of entangled states, the cluster states. The measurements are used to imprint a quantum logic circuit on the state, thereby destroying its entanglement at the same time. Cluster states are thus one-way quantum computers and the measurements form the program.

DOI: 10.1103/PhysRevLett.86.5188 PACS numbers: 03.67.Lx, 03.65.Ud

One-way quantum computation

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- Is realized using only local conditioned projective measurements applied to a highly entangled state called the cluster state,
- as it is entirely based on local measurements, instead of unitary evolution, the computation is inherently irreversible in time,
- the technical requirements for the one-way quantum computation can be much simpler than those for the standard circuit model.

One-way quantum computation

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PRL **95,** 210502 (2005) PHYSICAL REVIEW LETTERS week ending 18 NOVEMBER 2005

Experimental Analysis of a Four-Qubit Photon Cluster State

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Rupert Ursin,[4] and Harald Weinfurter[1,2]
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(Received 29 June 2005; published 16 November 2005)

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Experimental entanglement of six photons in graph states

CHAO-YANG LU[1*], XIAO-QI ZHOU[1], OTFRIED GÜHNE[2], WEI-BO GAO[1], JIN ZHANG[1], ZHEN-SHENG YUAN[1], ALEXANDER GOEBEL[3], TAO YANG[1] AND JIAN-WEI PAN[1,3*]

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One-way quantum computation

One-way quantum computation

PRL 103, 160401 (2009) PHYSICAL REVIEW LETTERS week ending 16 OCTOBER 2009

Experimental Entanglement and Nonlocality of a Two-Photon Six-Qubit Cluster State

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LETTERS

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nature physics

Optical one-way quantum computing with a simulated valence-bond solid

Rainer Kaltenbaek[1★†], Jonathan Lavoie[1†], Bei Zeng[2,3], Stephen D. Bartlett[4] and Kevin J. Resch[1★]

One-way quantum computation

The Cluster state

One-way quantum computation

The Cluster state

$$|\Phi_{ini}\rangle = |\psi_{\text{in}}\rangle_1 \otimes |+\rangle_2 \otimes |+\rangle_3 \otimes |+\rangle_4 \otimes |+\rangle_5$$

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$$|+\rangle_n = \frac{1}{\sqrt{2}} \left(|0\rangle_n + |1\rangle_n \right)$$

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$$\hat{H} = \hbar g \sum_{a,a'} f\left(a - a'\right) \frac{1 + \hat{\sigma}_z^a}{2} \frac{1 - \hat{\sigma}_z^{a'}}{2}$$

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$$\hat{H} = \hbar g \sum_{a,a'} f\left(a - a'\right) \frac{1 + \hat{\sigma}_z^a}{2} \frac{1 - \hat{\sigma}_z^{a'}}{2}$$

for a linear chain whose interaction is between first neighbors:

$$f\left(a - a'\right) = \delta_{a+1,a'}$$

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$$\hat{H}_{int} = -\frac{\hbar g}{4} \sum_{a,a'} f\left(a - a'\right) \hat{\sigma}_z^a \hat{\sigma}_z^{a'}$$

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$$\hat{S} = \exp\left(-\frac{i}{\hbar} \hat{H}_{int} \tau \right)$$

One-way quantum computation

The Cluster state

The Cluster state

$$
\begin{aligned}
S|\Phi_{ini}\rangle &= \frac{1}{2}\,|\psi_{\text{in}}\rangle_1\,|0\rangle_2|-\rangle_3|0\rangle_4|-\rangle_5 \\
&- \frac{1}{2}\,|\psi_{\text{in}}\rangle_1\,|0\rangle_2|+\rangle_3|1\rangle_4|+\rangle_5 \\
&- \frac{1}{2}\,|\psi^*_{\text{in}}\rangle_1\,|1\rangle_2|+\rangle_3|0\rangle_4|-\rangle_5 \\
&+ \frac{1}{2}\,|\psi^*_{\text{in}}\rangle_1\,|1\rangle_2|-\rangle_3|1\rangle_4|+\rangle_5
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$$

One-way quantum computation

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&- \frac{1}{2}|\psi^*_{\rm in}\rangle_1 |1\rangle_2|+\rangle_3|0\rangle_4|-\rangle_5 \\
&+ \frac{1}{2}|\psi^*_{\rm in}\rangle_1 |1\rangle_2|-\rangle_3|1\rangle_4|+\rangle_5
\end{aligned}
$$

where $|\psi^*_{\rm in}\rangle_1 = \sigma_z^{(1)}|\psi_{\rm in}\rangle_1 = \alpha|0\rangle_1 - \beta|1\rangle_1$ and $|-\rangle_n = \frac{1}{\sqrt{2}}(|0\rangle_n - |1\rangle_n)$

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The computation

$$
\mathcal{B}_j(\phi_j) = \left\{ \frac{|0\rangle_j + e^{i\phi_j}|1\rangle_j}{\sqrt{2}}, \frac{|0\rangle_j - e^{i\phi_j}|1\rangle_j}{\sqrt{2}} \right\}
$$

Example: How to implement a BIT-FLIP

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$$\mathcal{B}_1\left(0\right)=\frac{|0\rangle_1+|1\rangle_1}{\sqrt{2}}=|+\rangle_1$$

$$\mathcal{B}_2\left(-\pi\right)=\frac{|0\rangle_2+e^{-i\pi}|1\rangle_2}{\sqrt{2}}=\frac{|0\rangle_2-|1\rangle_2}{\sqrt{2}}=|-\rangle_2$$

$$\mathcal{B}_3\left(0\right)=\frac{|0\rangle_3+|1\rangle_3}{\sqrt{2}}=|+\rangle_3$$

$$\mathcal{B}_4\left(0\right)=\frac{|0\rangle_4+|1\rangle_4}{\sqrt{2}}=|+\rangle_4$$

One-way quantum computation

Example: How to implement a BIT-FLIP

$$\mathcal{B}_1(0) = \frac{|0\rangle_1 + |1\rangle_1}{\sqrt{2}} = |+\rangle_1$$

$$\mathcal{B}_2(-\pi) = \frac{|0\rangle_2 + e^{-i\pi}|1\rangle_2}{\sqrt{2}} = \frac{|0\rangle_2 - |1\rangle_2}{\sqrt{2}} = |-\rangle_2$$

$$\mathcal{B}_3(0) = \frac{|0\rangle_3 + |1\rangle_3}{\sqrt{2}} = |+\rangle_3$$

$$\mathcal{B}_4(0) = \frac{|0\rangle_4 + |1\rangle_4}{\sqrt{2}} = |+\rangle_4$$

$$\hat{\Pi}_1(0) = |+\rangle_1\langle+|$$

$$\hat{\Pi}_2(-\pi) = |-\rangle_2\langle-|$$

$$\hat{\Pi}_3(0) = |+\rangle_3\langle+|$$

$$\hat{\Pi}_4(0) = |+\rangle_4\langle+|$$

One-way quantum computation

Example: How to implement a BIT-FLIP

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So, we have four projectors: $\hat{\Pi}_1 = |+\rangle_1\langle+|$, $\hat{\Pi}_2 = |-\rangle_2\langle-|$, $\hat{\Pi}_3 = |+\rangle_3\langle+|$, and $\hat{\Pi}_4 = |+\rangle_4\langle+|$

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Our input and expected output are:

$$|\psi_{\text{in}}\rangle_1 = |0\rangle_1 \Rightarrow |\psi_{\text{out}}\rangle_5 = |1\rangle_5$$

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We begin with the following disentangled state:

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We begin with the following disentangled state:

$$|\Phi_{ini}\rangle = |0\rangle_1 \otimes |+\rangle_2 \otimes |+\rangle_3 \otimes |+\rangle_4 \otimes |+\rangle_5$$

The Cluster State is given by:

$$\begin{aligned} S|\Phi_{ini}\rangle &= \frac{1}{2}|0\rangle_1 |0\rangle_2|-\rangle_3|0\rangle_4|-\rangle_5 \\ &- \frac{1}{2}|0\rangle_1 |0\rangle_2|+\rangle_3|1\rangle_4|+\rangle_5 \\ &- \frac{1}{2}|0\rangle_1 |1\rangle_2|+\rangle_3|0\rangle_4|-\rangle_5 \\ &+ \frac{1}{2}|0\rangle_1 |1\rangle_2|-\rangle_3|1\rangle_4|+\rangle_5 \end{aligned}$$

Example: How to implement a BIT-FLIP

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Applying these four measurements in the Cluster state we obtain the following state:

$$\hat{\Pi}_4\hat{\Pi}_3\hat{\Pi}_2\hat{\Pi}_1 S|\Phi_{ini}\rangle = |+\rangle_1|-\rangle_2|+\rangle_3|+\rangle_4|0\rangle_5$$

Outline

Exact dissipative dynamics

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PHYSICAL REVIEW A, VOLUME 65, 032326

Decoherence of quantum registers

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(Received 8 May 2001; revised manuscript received 19 July 2001; published 1 March 2002)

Exact dissipative dynamics

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- They consider a N-qubit system interacting with a common and a independent dephasing environment.

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- The symmetry of the Hamiltonian allows the completely determination of the propagator.

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- They consider a N-qubit system interacting with a common and a independent dephasing environment.
- The symmetry of the Hamiltonian allows the completely determination of the propagator.
- The dynamical evolution of a N-qubit state is calculated exactly.

Exact dissipative dynamics

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Decoherence of quantum registers

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- They consider a N-qubit system interacting with a common and a independent dephasing environment.
- The symmetry of the Hamiltonian allows the completely determination of the propagator.
- The dynamical evolution of a N-qubit state is calculated exactly.
- The solution of the particular case of common dephasing environment allows an analysis of different time and temperature scales.

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The time evolution operator

$$U_I(t) = \hat{T}\exp\left(-\frac{i}{\hbar}\int_0^t \tilde{H}_I(t')\,dt'\right)$$

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$$\rho^{Q}_{\{i_n,j_n\}}(t) = \langle i_1, i_2, \ldots, i_N | \rho^{Q}(t) | j_1, j_2, \ldots, j_N \rangle$$

$$i_n, j_n = \pm 1$$

j_n are the eigenvalues of the $\sigma_z^{(n)}$ Pauli operator associated with $|0\rangle_n$ and $|1\rangle_n$,

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The evolution of the element $\rho^{Q}_{\{i_n,j_n\}}$:

$$\rho^{Q}_{\{i_n,j_n\}}(t) = \exp\left\{-\Gamma(t,T)\left[\sum_{n=1}^{N}(i_n - j_n)\right]^2\right\}$$
$$\times \quad \exp\left\{i\Theta(t)\left[\left(\sum_{n=1}^{N} i_n\right)^2 - \left(\sum_{n=1}^{N} j_n\right)^2\right]\right\}\rho^{Q}_{\{i_n,j_n\}}(0) \qquad (1)$$

Exact dissipative dynamics

In the continuum limit

$$\Gamma\left(t,T\right)=\int d\omega J\left(\omega\right)c\left(\omega,t\right)\coth\left(\frac{\hbar\omega}{2k_{B}T}\right), \tag{2}$$

$$\Theta\left(t\right)=\int d\omega J\left(\omega\right)s\left(\omega,t\right), \tag{3}$$

with $c\left(\omega,t\right)=\frac{1-\cos(\omega t)}{\omega^{2}}$ and $s\left(\omega,t\right)=\frac{\omega t-\sin(\omega t)}{\omega^{2}}$

In our model we assume an ohmic spectral density,

$$J\left(\omega\right)=\eta\omega e^{-\omega/\omega_{c}}, \tag{4}$$

η is a dimensionless proportionality constant that characterizes the coupling strength between the system and the environment.
The result of the integration is also well-known and reads:

$$\Theta\left(t\right)=\eta\omega_{c}t-\eta\arctan\left(\omega_{c}t\right) \tag{5}$$

$$\Gamma\left(t,T\right)=\eta\ln(1+\omega_{c}^{2}t^{2})+\eta\ln\left(\frac{\beta\hbar}{\pi t}\sinh\frac{\pi t}{\beta\hbar}\right) \tag{6}$$

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$\tau_c < t < \tau_T$

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Thermal regime: $t > \tau_T$

for a sufficiently high-temperature environment, i.e., $\hbar\omega_c \gtrsim k_B T$: $\Gamma(t,T) \approx \eta \omega_T t$

Outline

Oscillatory fidelity dynamics

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The oscillatory term in the evolution of the element $\rho^{Q}_{\{i_n,j_n\}}$:

$$\exp\left\{i\Theta\left(t\right)\left[\left(\sum_{n=1}^{N} i_n\right)^2 - \left(\sum_{n=1}^{N} j_n\right)^2\right]\right\}\rho^{Q}_{\{i_n,j_n\}}\left(0\right)$$

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$\Rightarrow$ The oscillatory behavior of the fidelity dynamics of qubits interacting with a common dephasing environment is strongly dependent upon the initial condition

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- Thus, if the initial state of the N-qubit system is a coherent superposition of eigenstates of the $\sigma_z^{(T)}$ operator, whose eigenvalues are equal in modulus, the condition $\left|\sum_{n=1}^{N} i_n\right| = \left|\sum_{n=1}^{N} j_n\right|$ is automatically satisfied and the fidelity dynamics does not oscillate at all.

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The necessary condition for the non-monotonical behavior of the fidelity dynamics

$$\left|\sum_{n=1}^{N} i_n\right| \neq \left|\sum_{n=1}^{N} j_n\right|$$

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- We assume, for simplicity, that the first measurement is applied at $t_0 = 0$, then:

$$\begin{aligned}
|\psi\rangle_{2,\ldots,5}(0) &= \frac{\alpha+\beta}{2\sqrt{2}}|0\rangle_2|-\rangle_3|0\rangle_4|-\rangle_5 \\
&- \frac{\alpha+\beta}{2\sqrt{2}}|0\rangle_2|+\rangle_3|1\rangle_4|+\rangle_5 \\
&- \frac{\alpha-\beta}{2\sqrt{2}}|1\rangle_2|+\rangle_3|0\rangle_4|-\rangle_5 \\
&+ \frac{\alpha-\beta}{2\sqrt{2}}|1\rangle_2|-\rangle_3|1\rangle_4|+\rangle_5.
\end{aligned}$$

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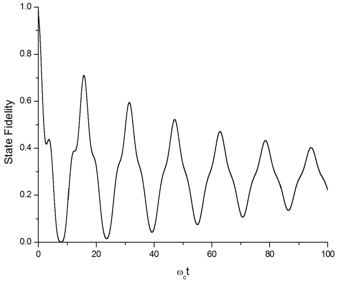
The state fidelity:

- For the input state $|\phi_{\text{in}}\rangle_1 = |0\rangle_1$ $(\alpha = 1)$
- In the quantum regime
- with $\eta = 1/1000$, $\omega_c = 100$, and $\omega_T = 1$



$$
\begin{aligned}
F_{|\psi\rangle_{2,\ldots,5}}(t) = {} & \frac{3}{32}e^{-16\Gamma(t,T)}\cos\left[16\Theta(t)\right] + \frac{3}{8}e^{-4\Gamma(t,T)}\cos\left(4\Theta(t)\right) + \left[\frac{1}{16} - \frac{1}{32}\left(\alpha^*\beta + \alpha\beta^*\right)^2\right]e^{-36\Gamma(t,T)}\cos\left(12\Theta(t)\right) \\
& + \left[\frac{1}{16} + \frac{1}{32}\left(\alpha^*\beta + \alpha\beta^*\right)^2\right]e^{-4\Gamma(t,T)}\cos\left(12\Theta(t)\right) + \left[\frac{1}{128} - \frac{1}{128}\left(\alpha^*\beta + \alpha\beta^*\right)^2\right]e^{-64\Gamma(t,T)} \\
& + \left[\frac{1}{8} - \frac{1}{32}\left(\alpha^*\beta + \alpha\beta^*\right)^2\right]e^{-16\Gamma(t,T)} + \frac{5}{128}\left(\alpha^*\beta + \alpha\beta^*\right)^2 + \frac{35}{128}.
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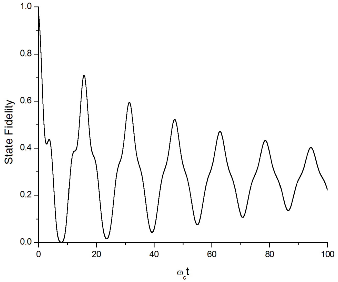

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&+ \left[\frac{1}{16} + \frac{1}{32}\left(\alpha^*\beta + \alpha\beta^*\right)^2\right]e^{-4\Gamma(t,T)}\cos\left(12\Theta(t)\right) + \left[\frac{1}{128} - \frac{1}{128}\left(\alpha^*\beta + \alpha\beta^*\right)^2\right]e^{-64\Gamma(t,T)} \\
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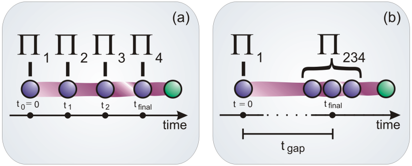
\end{aligned}
$$

With this in mind, what can we say about the fidelity of quantum computation in this peculiar regime?

Fidelity dynamics of an MBQC



The gate fidelity:

- At $t_0 = 0$ we consider that the five-qubit state is already entangled and each qubit is ready to be measured,
- besides, the first qubit is also projected at $t_0 = 0$.

The gate fidelity:

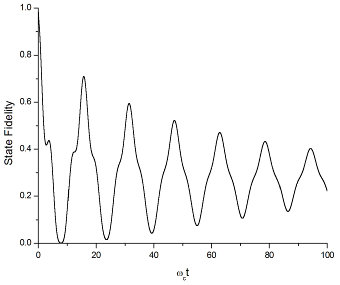
TWO SCENARIOS:

- In (a) we suppose that the three subsequent measurements are applied at different instants of time and the qubits evolve non-unitarilly between the measurements
- In (b), after wait a time gap, the other three subsequent measurements are made instantaneously at $t = t_{final}$

Fidelity dynamics of an MBQC: Measurements performed at different times

The NOT-GATE fidelity:

- $\Pi_2 = |-\rangle_2\langle -|$, $\Pi_3 = |+\rangle_3\langle +|$ and $\Pi_4 = |+\rangle_4\langle +|$
- $|\phi_{\text{in}}\rangle_1 = |0\rangle_1 \Rightarrow |\phi_{\text{out}}\rangle_5 = |1\rangle_5$
- $(t_1; t_2; t_{final})$
- $(6/\omega_c, 8/\omega_c, 10/\omega_c) \Rightarrow F = 35,4\%$
- $(14/\omega_c, 16/\omega_c, 18/\omega_c) \Rightarrow F = 53\%$
- $(15.2/\omega_c, 15.7/\omega_c, 16.2/\omega_c) \Rightarrow F = 84\%$
- $(15.5/\omega_c, 15.7/\omega_c, 15.9/\omega_c) \Rightarrow F = 90\%$



with $\eta = 1/1000$, $\omega_c = 100$, and $\omega_T = 1$

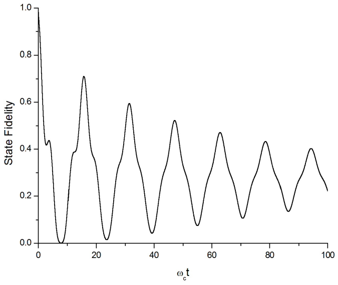
The NOT-GATE fidelity:

- $(7,8/\omega_c; 23,4/\omega_c; 39/\omega_c) \Rightarrow F = 50\%$
- $(15,7/\omega_c; 31,4/\omega_c; 47,1/\omega_c) \Rightarrow F = 76\%$

Fidelity dynamics of an MBQC: Measurements performed at different times

The HADAMARD-GATE fidelity:

- $\Pi_2 = |-, y\rangle_2\langle -, y|$,
  $\Pi_3 = |+, y\rangle_3\langle +, y|$, and
  $\Pi_4 = |+\rangle_4\langle +|$, where
  $|\pm, y\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm i|1\rangle)$
- $|\psi_{\text{in}}\rangle_1 = |0\rangle_1 \Rightarrow |\psi_{\text{out}}\rangle_5 = \frac{1}{\sqrt{2}}(|0\rangle_5 + |1\rangle_5)$
- $(6/\omega_c, 8/\omega_c, 10/\omega_c) \Rightarrow F = 39\%$
- $(14/\omega_c, 16/\omega_c, 18/\omega_c) \Rightarrow F = 52\%$
- $(15.5/\omega_c, 15.7/\omega_c, 15.9/\omega_c) \Rightarrow F = 85\%$



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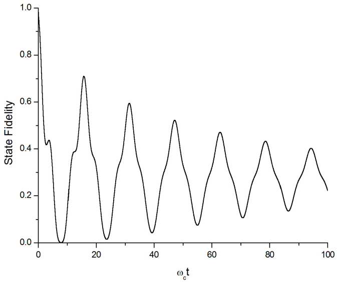
The HADAMARD-GATE fidelity:

- $(7, 8/\omega_c; 23, 4/\omega_c; 39/\omega_c) \Rightarrow F = 50\%$
- $(15, 7/\omega_c; 31, 4/\omega_c; 47, 1/\omega_c) \Rightarrow F = 76\%$

Fidelity dynamics of an MBQC: Measurements performed at different times

The PHASE-GATE fidelity:

- $\Pi_2 = |+\rangle_2\langle+|$, $\Pi_3 = |+,y\rangle_2\langle+,y|$, and $\Pi_4 = |+\rangle_4\langle+|$
- $|\psi_{\text{in}}\rangle_1 = \frac{1}{\sqrt{2}}\left(|0\rangle_1 + |1\rangle_1\right) \Rightarrow |\psi_{\text{out}}\rangle_5 = \frac{1}{\sqrt{2}}\left(|0\rangle_5 + i\,|1\rangle_5\right)$
- $(6/\omega_c, 8/\omega_c, 10/\omega_c) \Rightarrow F = 48\%$
- $(14/\omega_c, 16/\omega_c, 18/\omega_c) \Rightarrow F = 65\%$
- $(15.5/\omega_c, 15.7/\omega_c, 15.9/\omega_c) \Rightarrow F = 95\%$



with $\eta = 1/1000$, $\omega_c = 100$, and $\omega_T = 1$

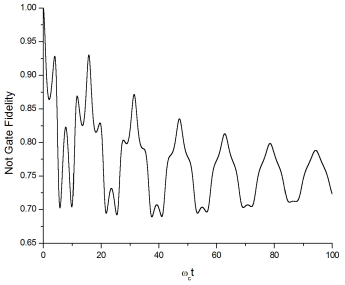
The PHASE-GATE fidelity:

- $(7,8/\omega_c; 23,4/\omega_c; 39/\omega_c) \Rightarrow F = 46\%$
- $(15,7/\omega_c; 31,4/\omega_c; 47,1/\omega_c) \Rightarrow F = 85\%$

Fidelity dynamics of an MBQC: Measurements performed at the same time

The NOT-GATE fidelity:

- t_{gap} is greater than $0,8/\omega_c$ (where the gate fidelity is 93%),
- $t_{gap} = 15,7/\omega_c$, when it reaches 93% again.
- Times such as $t_{gap} = 31,4/\omega_c$ or $t_{gap} = 47,1/\omega_c$, we still get a gate fidelity better than 80%,
- while at times such as $t_{gap} = 5,8/\omega_c$ we obtain a gate fidelity of 70%.

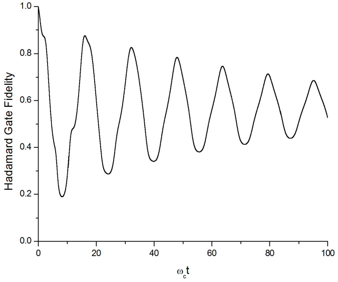


with $\eta = 1/1000$, $\omega_c = 100$, and $\omega_T = 1$

Fidelity dynamics of an MBQC: Measurements performed at the same time

The HADAMARD-GATE fidelity:

- Fidelities greater than 80% at times such as $t_{gap} = 15,7/\omega_c$, $t_{gap} = 31,4/\omega_c$ or $t_{gap} = 47,1/\omega_c$,
- Fidelities less than 40% at times such as $t_{gap} = 7,8/\omega_c$, $t_{gap} = 23,5/\omega_c$ or $t_{gap} = 39,2/\omega_c$.

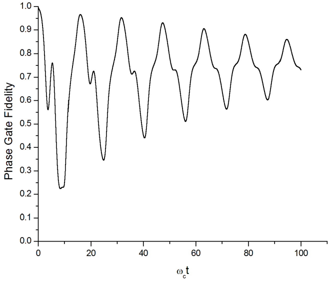


with $\eta = 1/1000$, $\omega_c = 100$, and $\omega_T = 1$

Fidelity dynamics of an MBQC: Measurements performed at the same time

The PHASE-GATE fidelity:

- At times like $t_{gap} = 15,9/\omega_c$, $t_{gap} = 31,6/\omega_c$ or $t_{gap} = 47,3/\omega_c$ we have a gate fidelity of 96%, 95% and 93%, respectively,
- while at times such as $t_{gap} = 8,4/\omega_c$, $t_{gap} = 24,8/\omega_c$ or $t_{gap} = 40,4/\omega_c$ we have a gate fidelity of 22%, 34% and 44%.



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Outline

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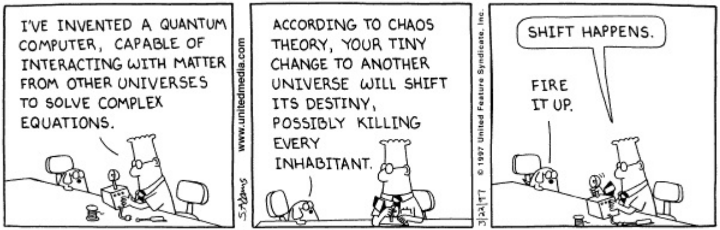
that is:

under the action of a common dephasing environment, this nonmonotonical time dependence can provide us with appropriate time intervals for the preservation of better computational fidelities.



One way quantum computers may be useful...

Thanks for your atention